





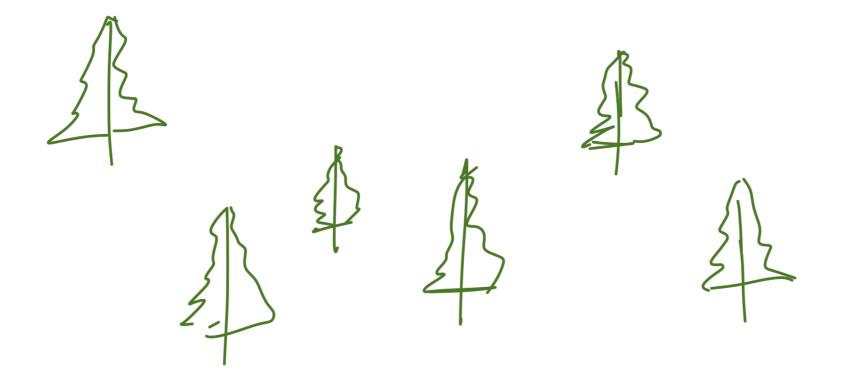


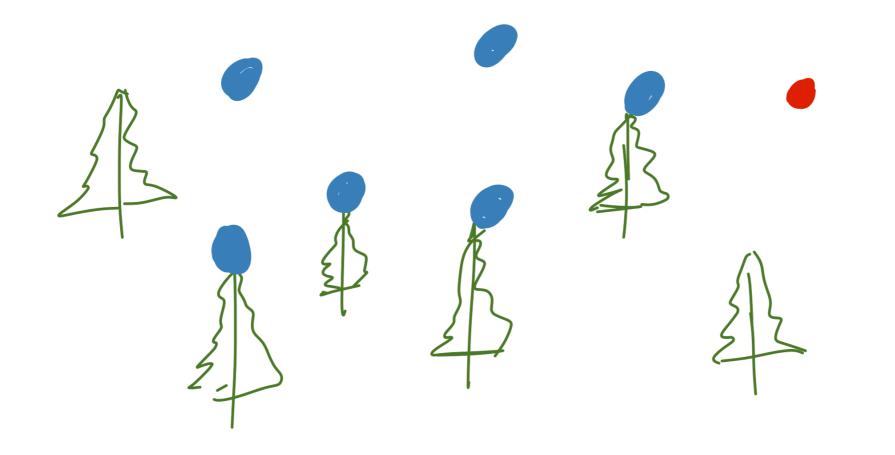
# Distributed Online Data Aggregation in Dynamic Graphs

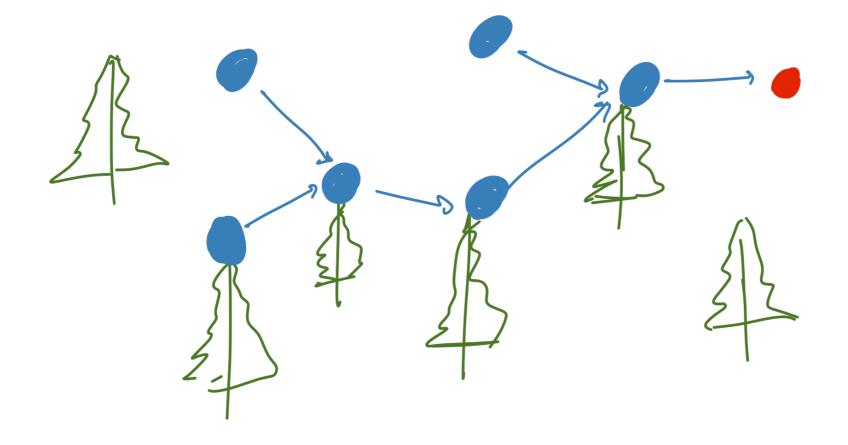
Quentin Bramas, Toshimitsu Masuzawa, and Sébastien Tixeuil

NETYS 2019, Marrakech, June, 21st

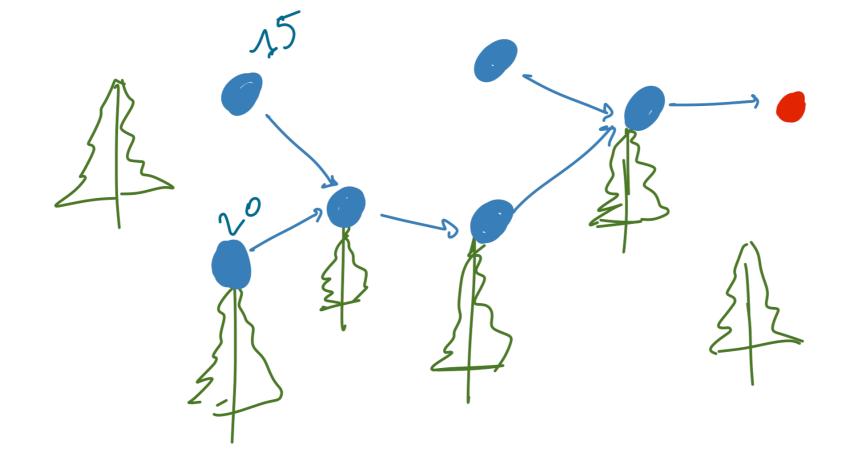
bramas@unistra.fr

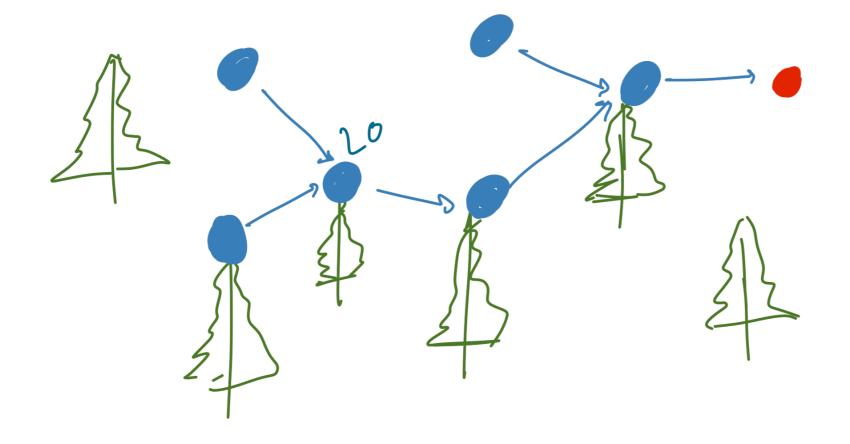


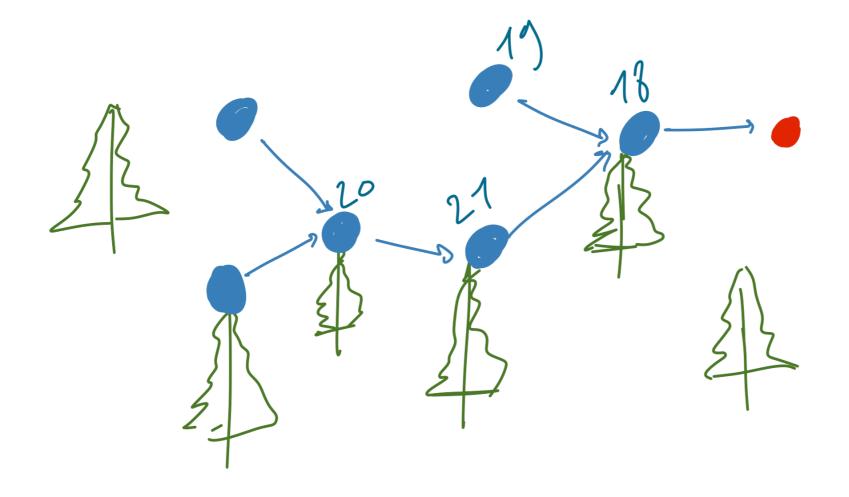


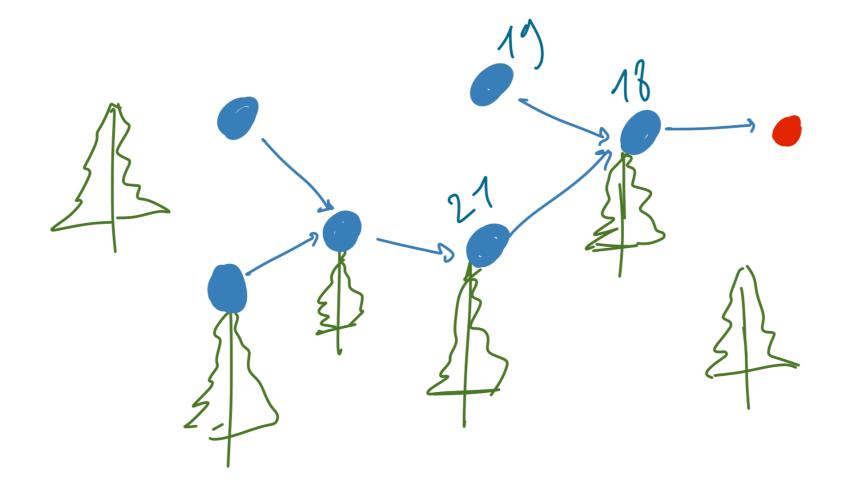


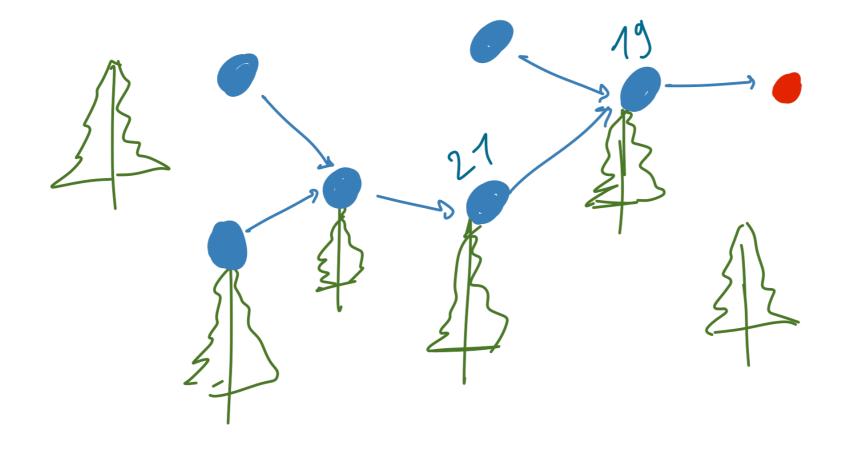
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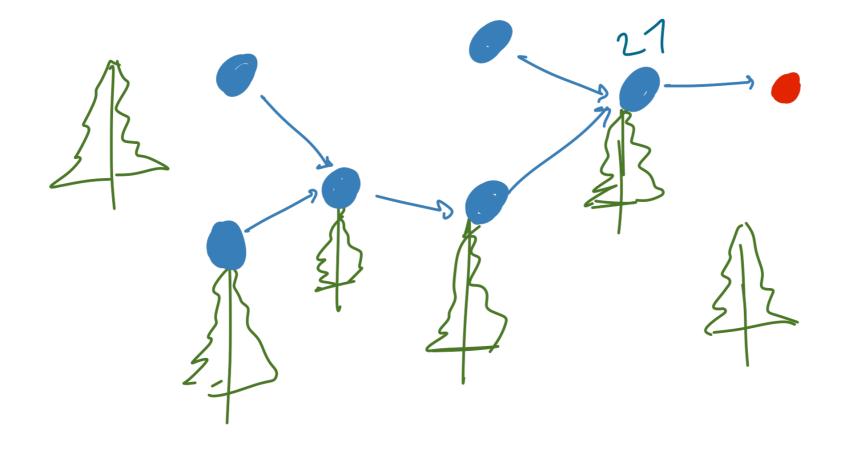


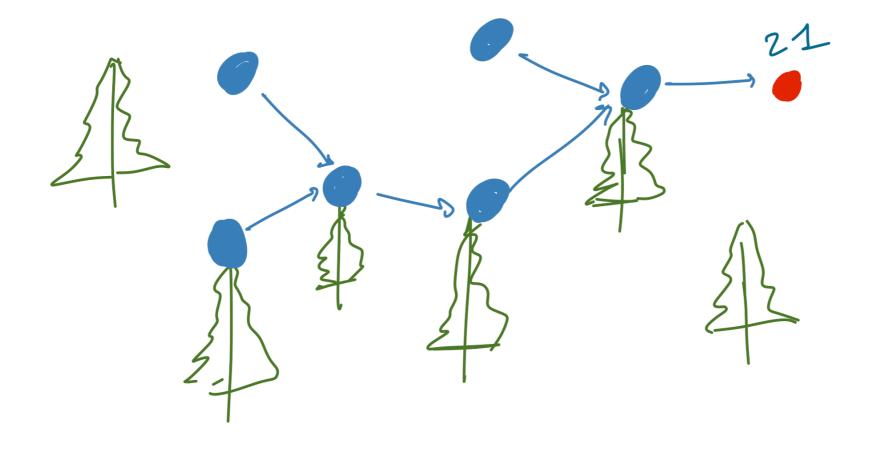


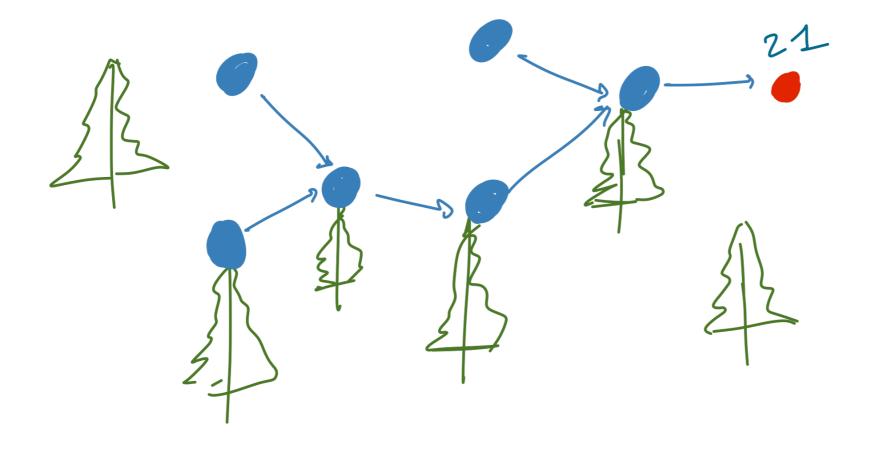




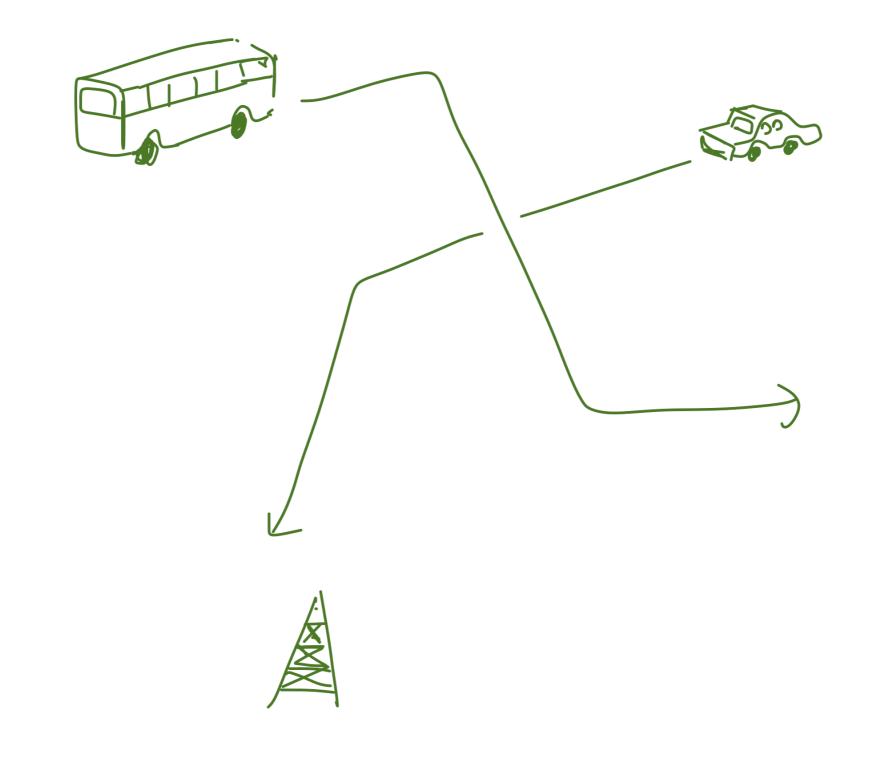


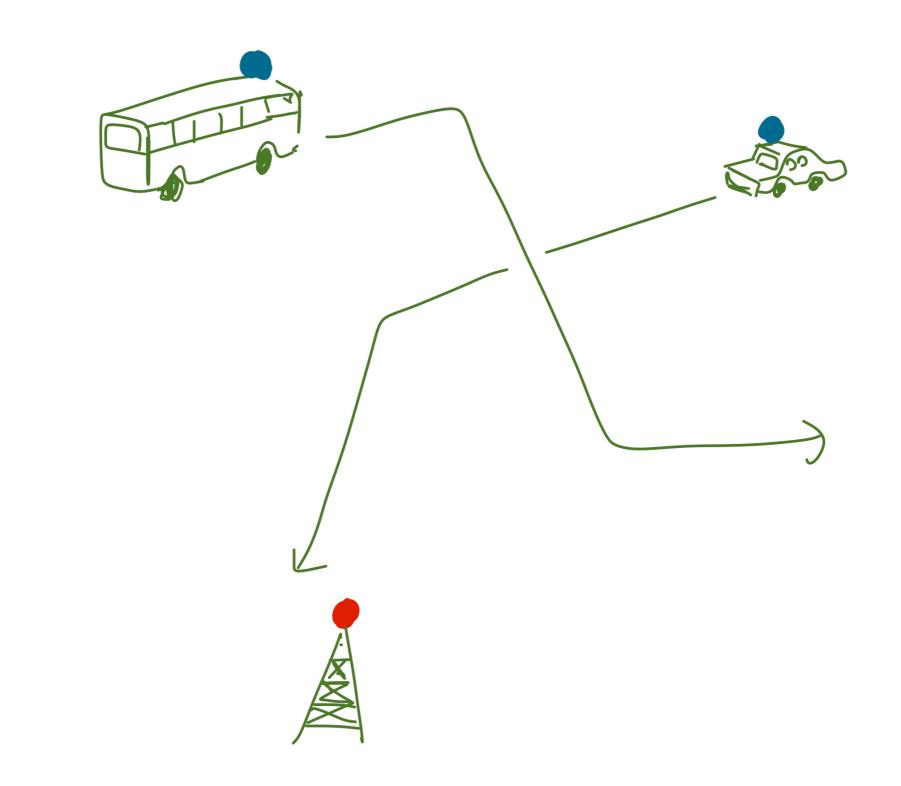


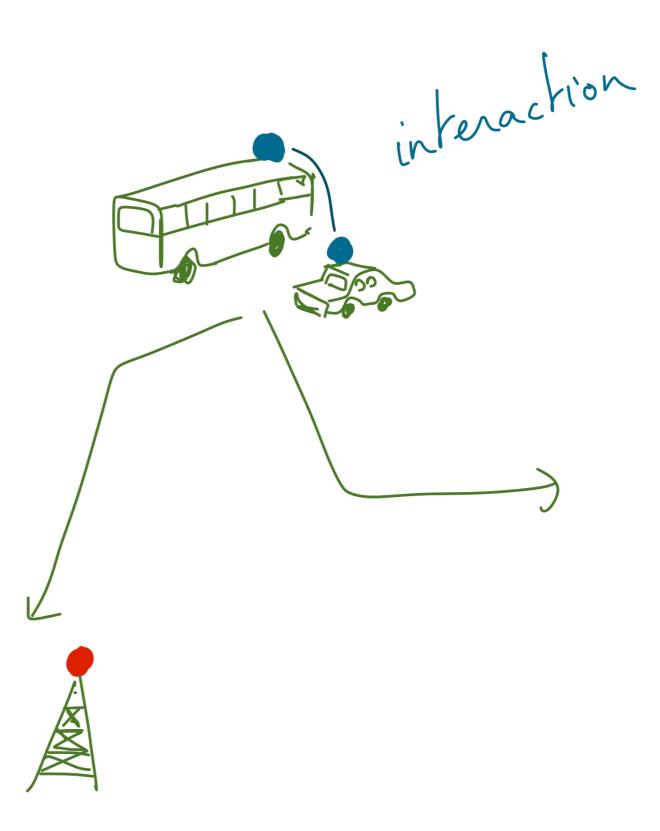


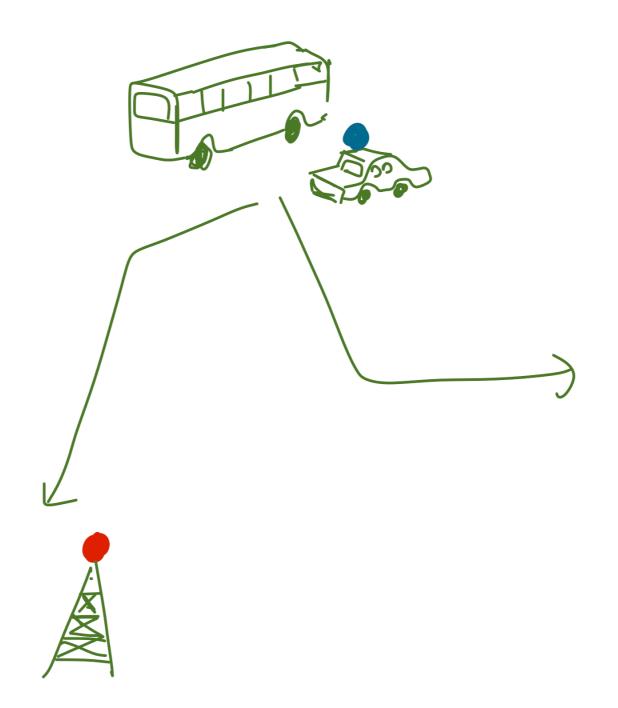


Each node transmits at most once. The goal: to minimize the duration







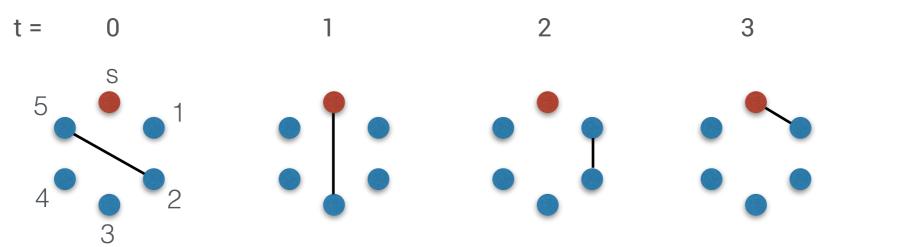




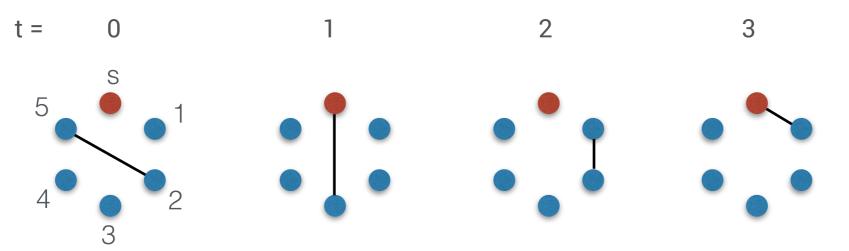


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#### We consider a dynamic network with pairwise interactions

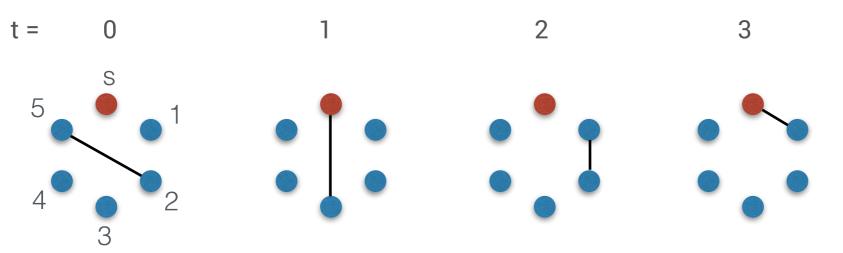


We consider a dynamic network with pairwise interactions



#### We consider a sequence of interactions

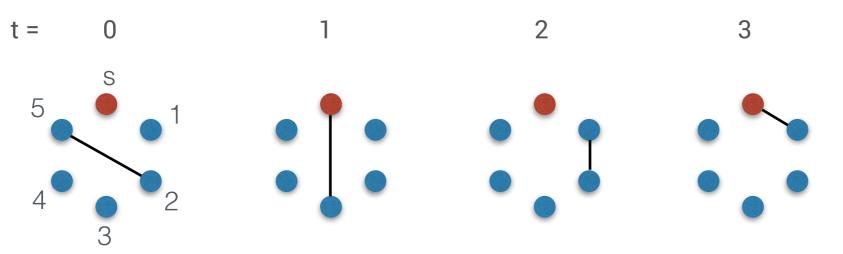
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We consider a sequence of interactions

A node can transmit only once

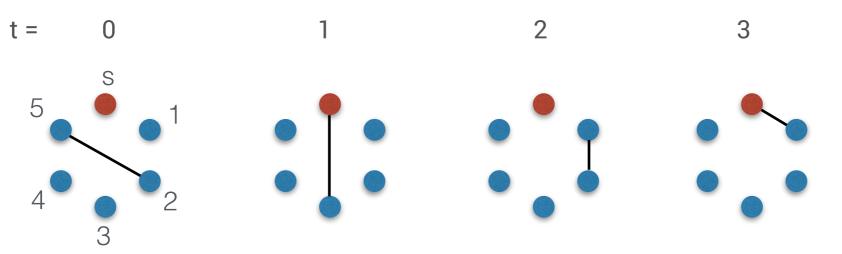
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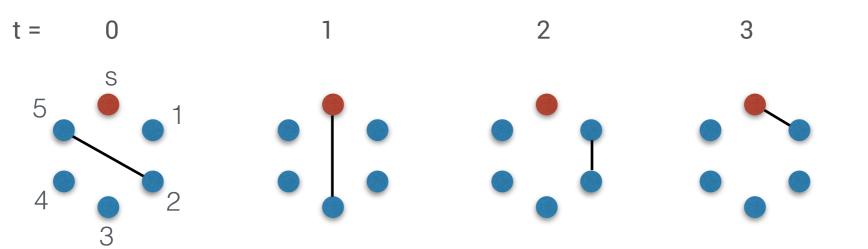
- A node can transmit only once
- Nodes may or may not have other information

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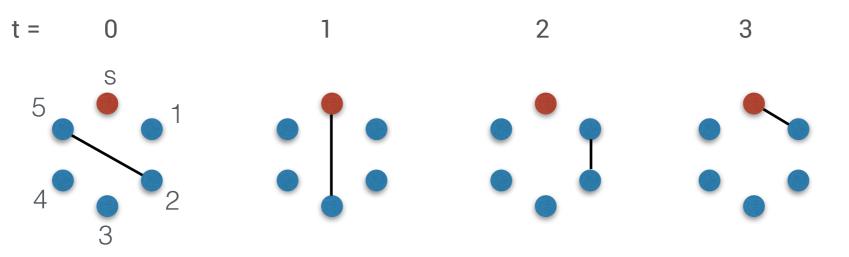
- We consider a sequence of interactions
- A node can transmit only once
- Nodes may or may not have other information
- The goal is to aggregate all the data with minimum duration

#### We consider a dynamic network with pairwise interactions



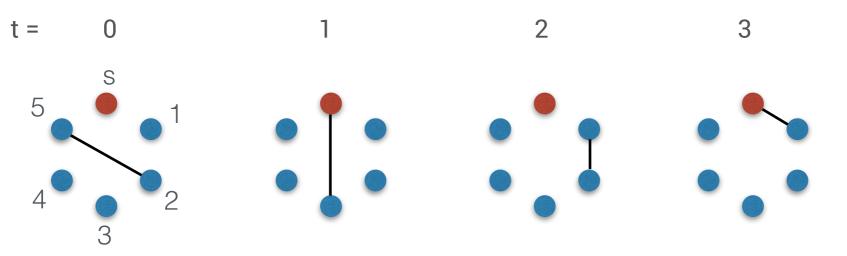
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#### We consider a dynamic network with pairwise interactions



A Distributed Online Data Aggregation (DODA) Algorithm answers the question: Which node transmits?

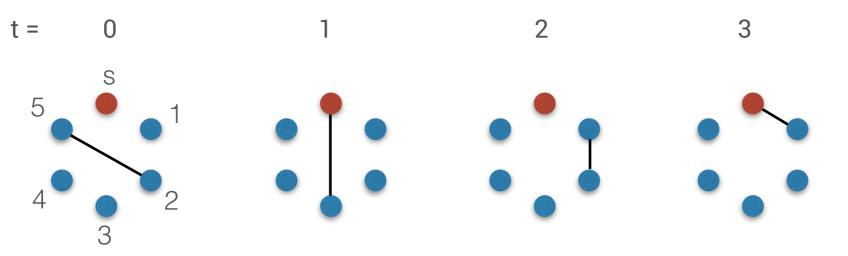
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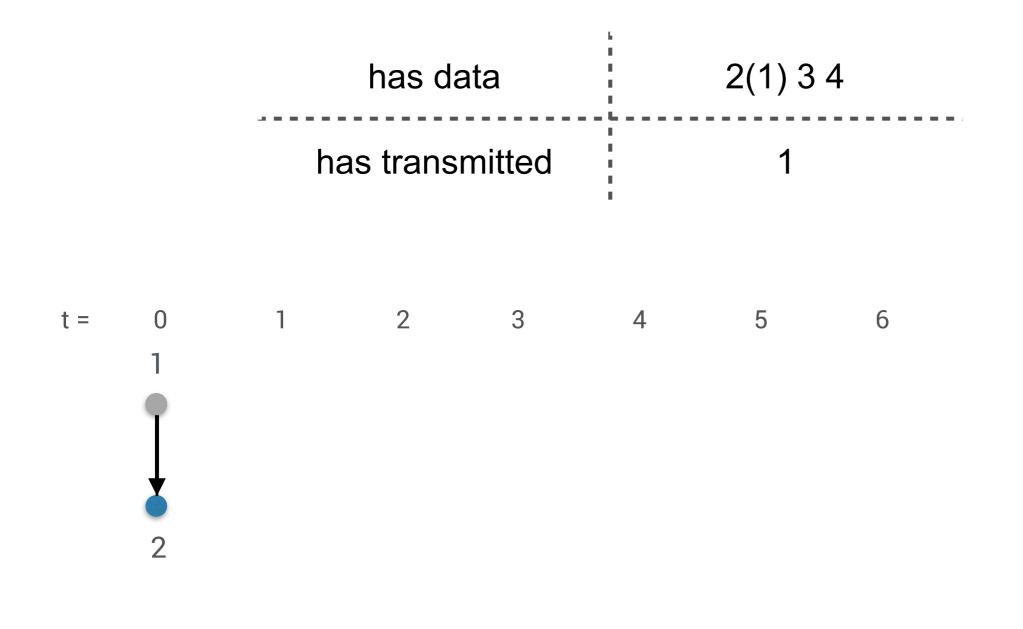
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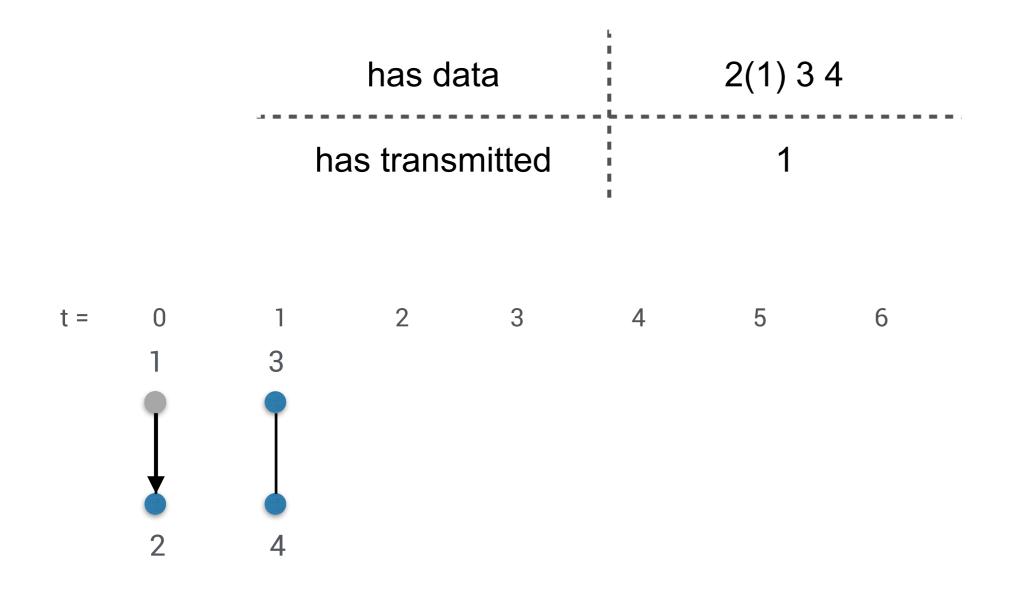


a transmits or b transmits or no one transmits

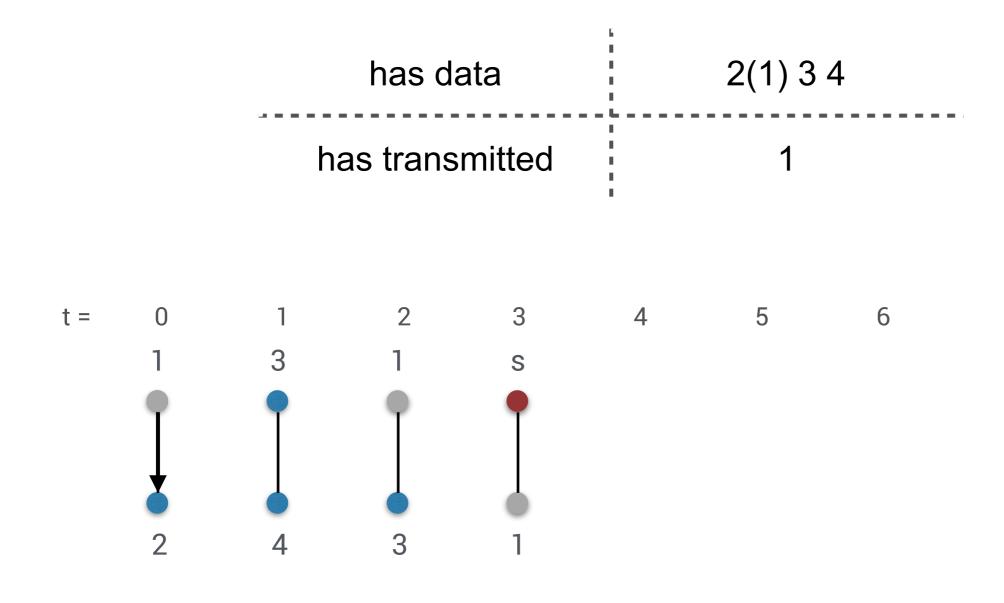
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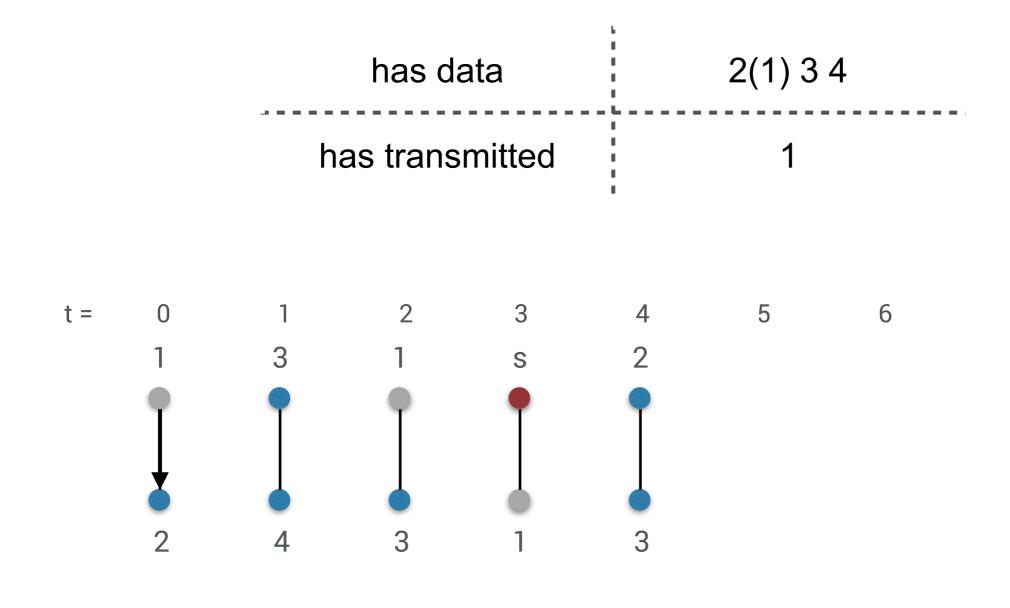




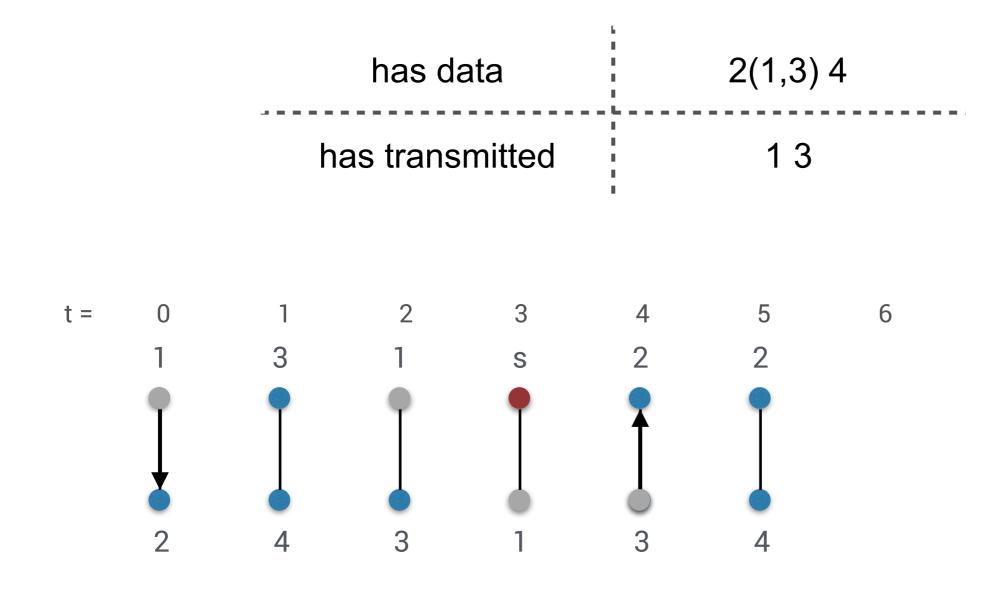


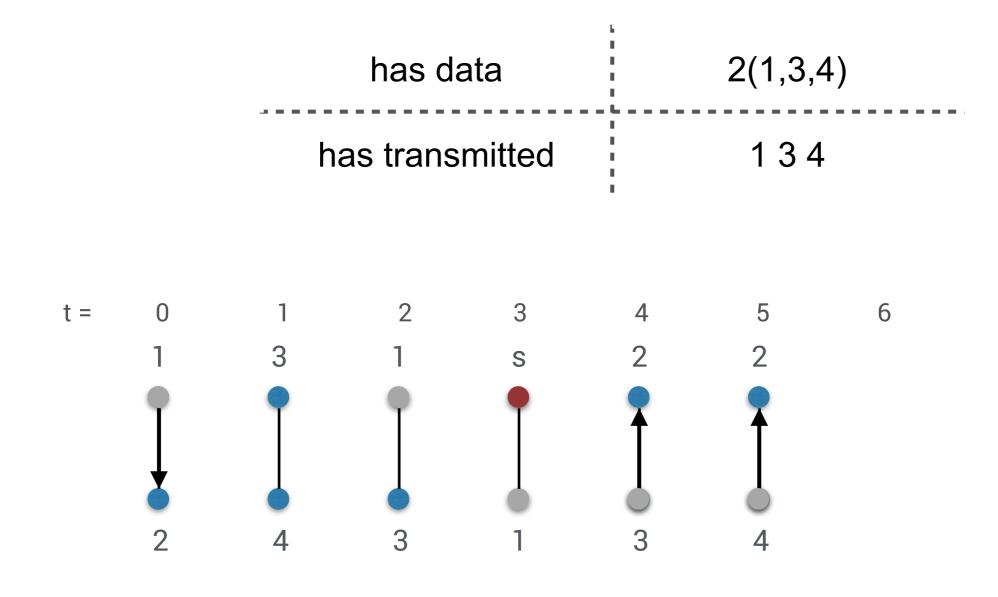


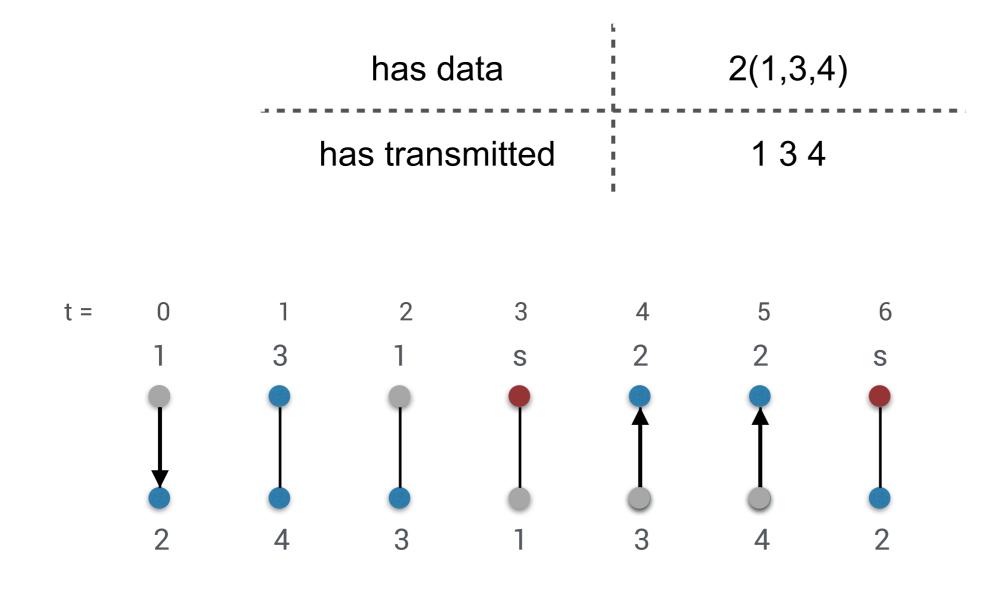


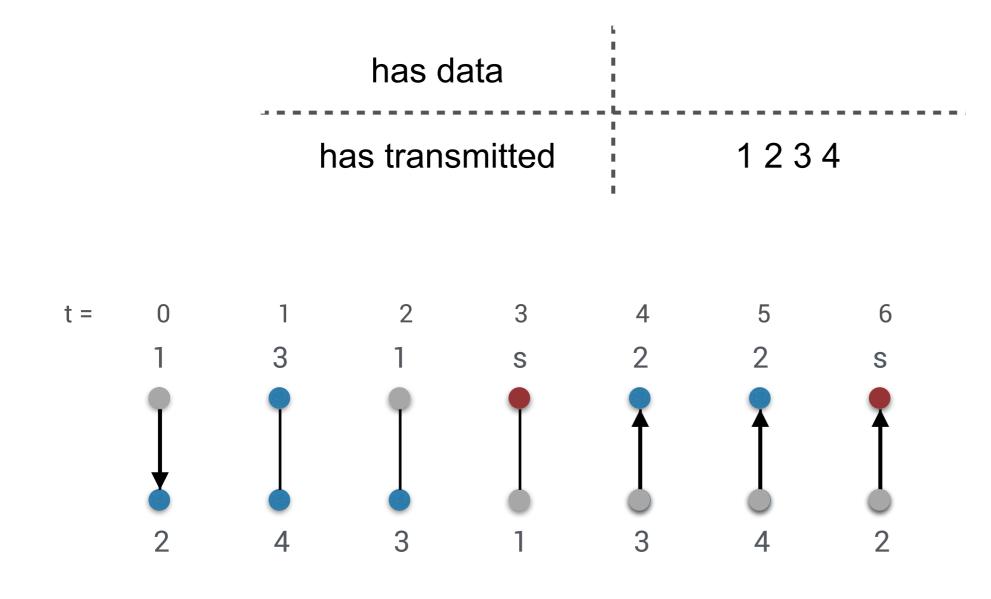












Is the duration of the aggregation is significant?

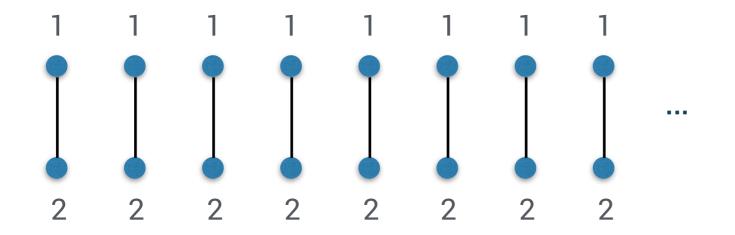
21

Is the duration of the aggregation is significant? No

21

Is the duration of the aggregation is significant? No

Even the offline optimal algorithm will struggle in the sequence:



22

Is the duration of the aggregation significant ? No

Is the ratio between the duration of the aggregation and the duration of the offline optimal algorithm significant ?

22

Is the duration of the aggregation significant ? No

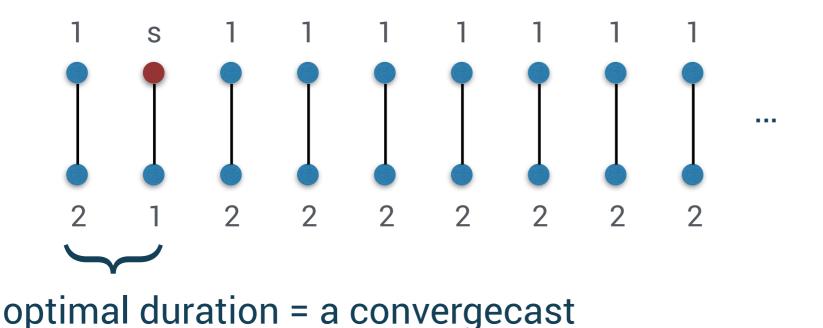
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22

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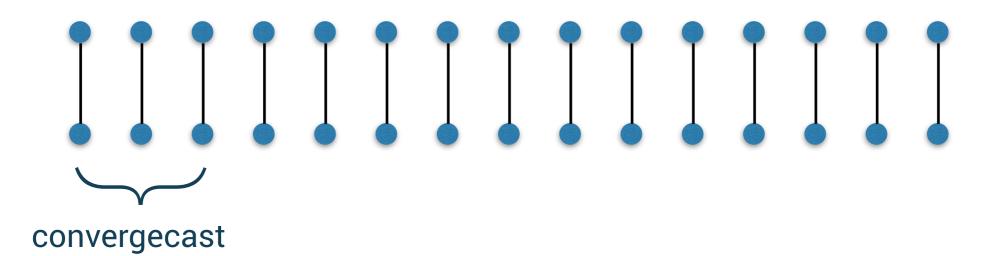


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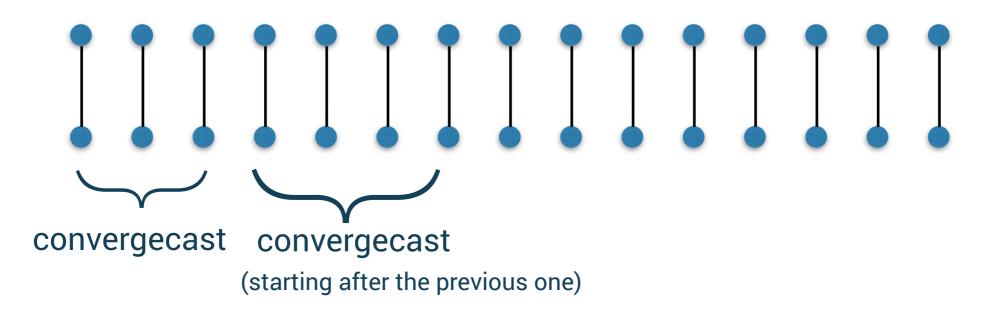
**Definition** of  $cost_I(A)$ , for an algorithm A in a sequence I:

# 

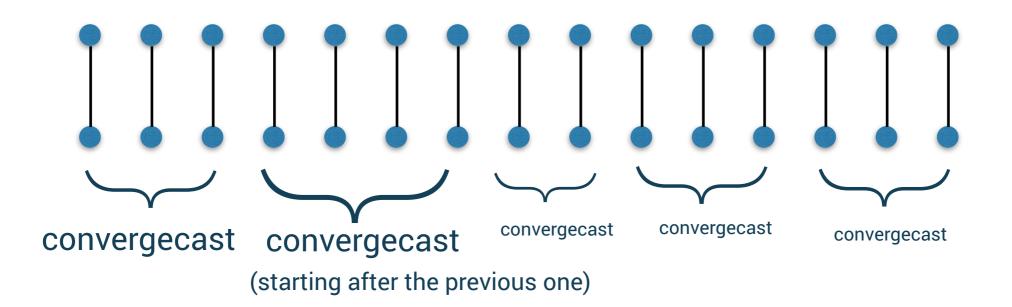
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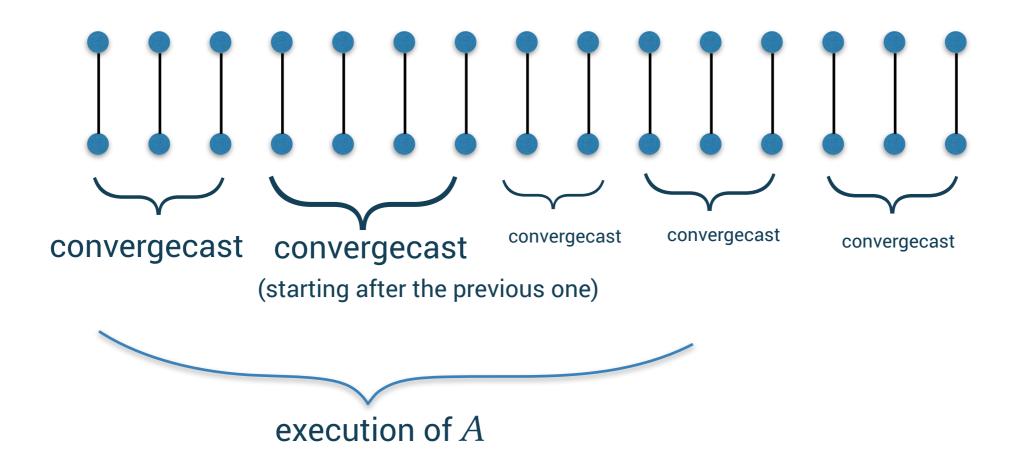
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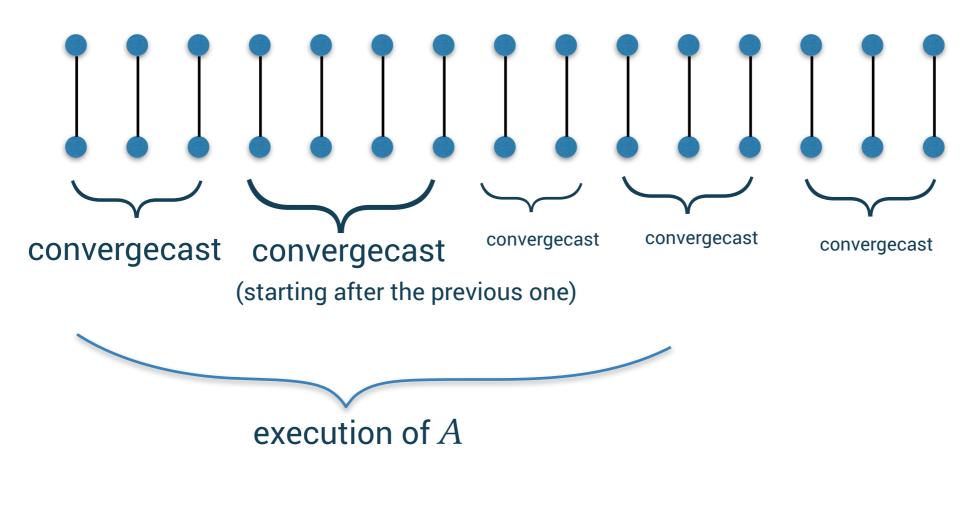


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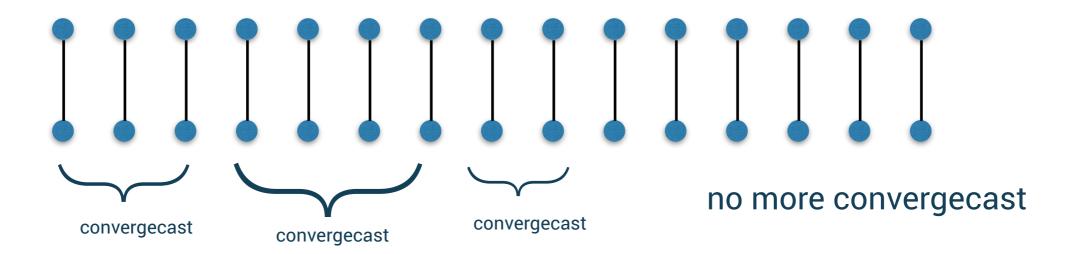
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**Definition** of  $cost_I(A)$ , for an algorithm A in a sequence I:



 $cost_I(A) = 4$ 

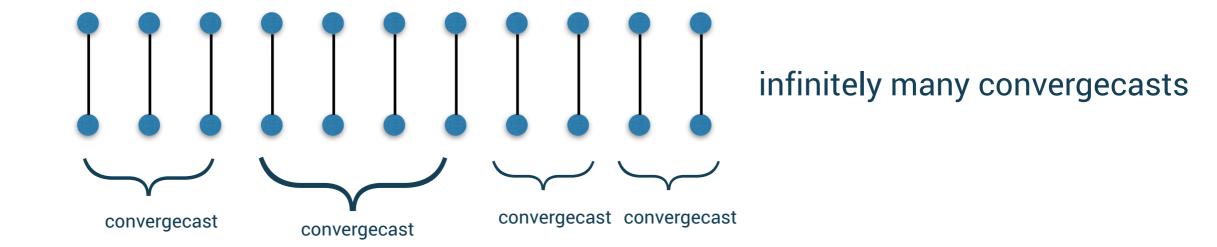
**Definition** of the  $cost_I(A)$  function, for an algorithm A in a sequence I:



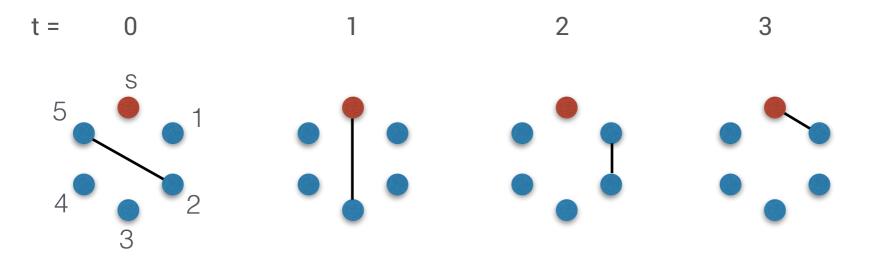
A does not terminate  $cost_I(A) = 4$ 

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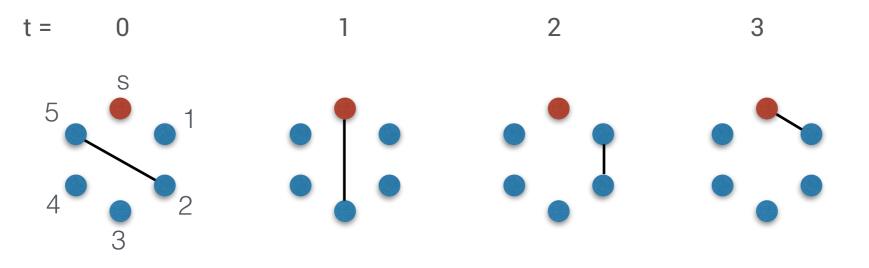
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A does not terminate  $cost_I(A) = \infty$ 

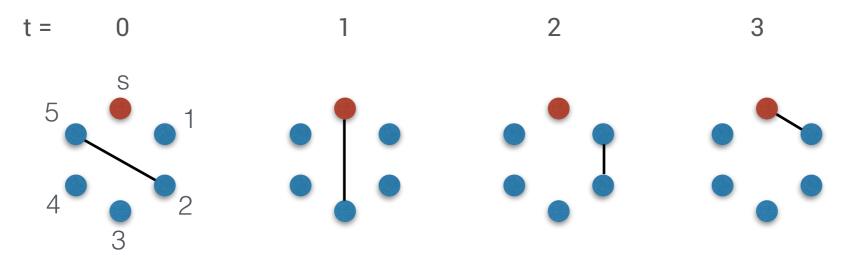


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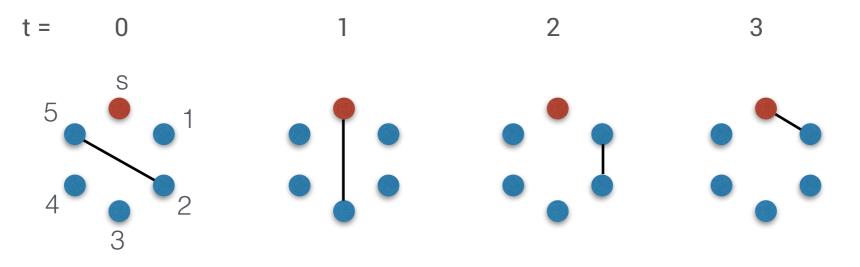


Who generates the sequence?

. . .



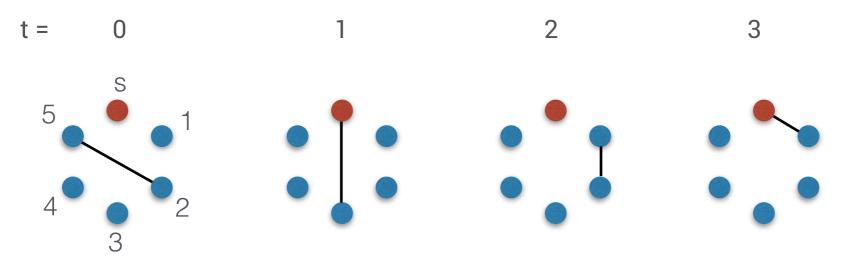
Who generates the sequence? An adversary



Who generates the sequence? An adversary

Three adversaries:

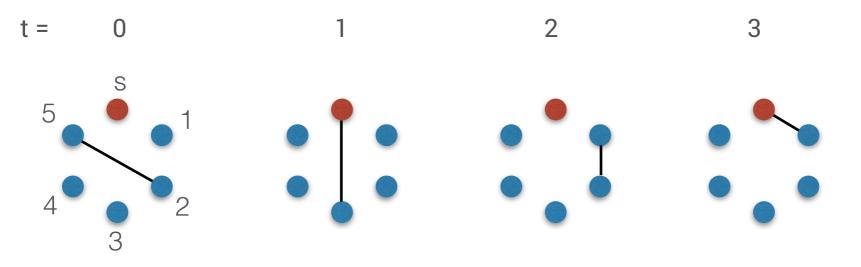
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Who generates the sequence? An adversary

Three adversaries:

- **Online Adaptive**: generates the next interaction based on what happened in the past

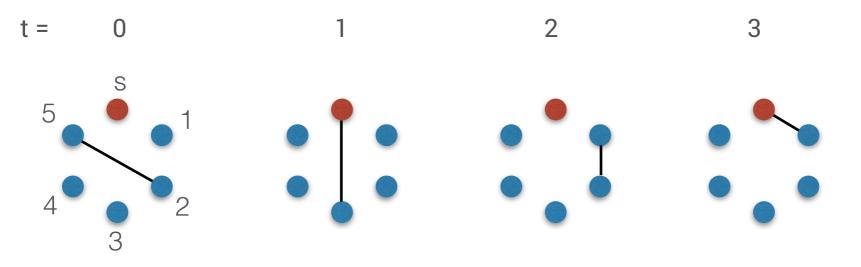


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Who generates the sequence? An adversary

Three adversaries:

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- **Oblivious**: generates the sequence before the execution of the algorithm

- Randomized: each interaction is chosen uniformly at random.

- **Online Adaptive**: generates the next interaction based on what the algorithm decides in the current interaction

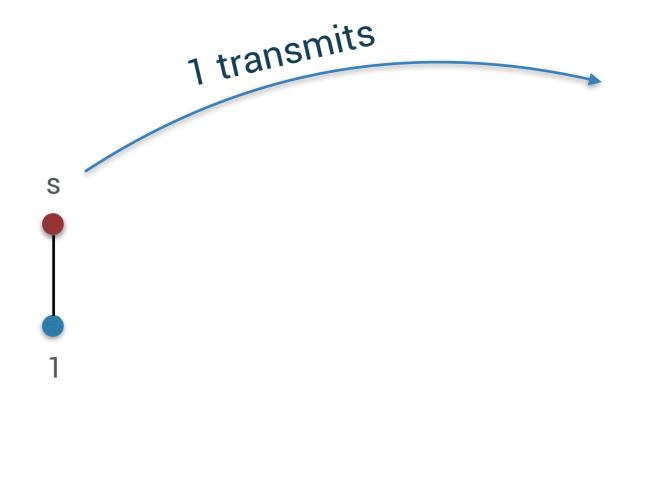
Let A be a DODA

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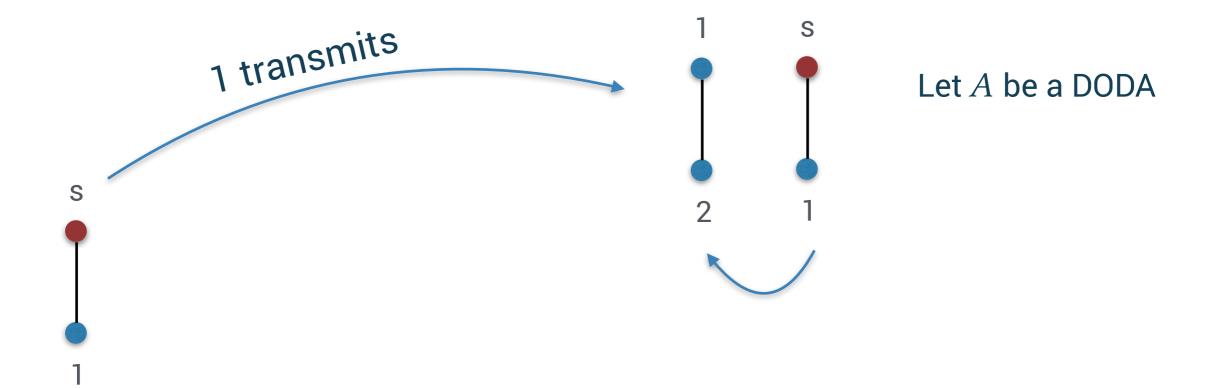
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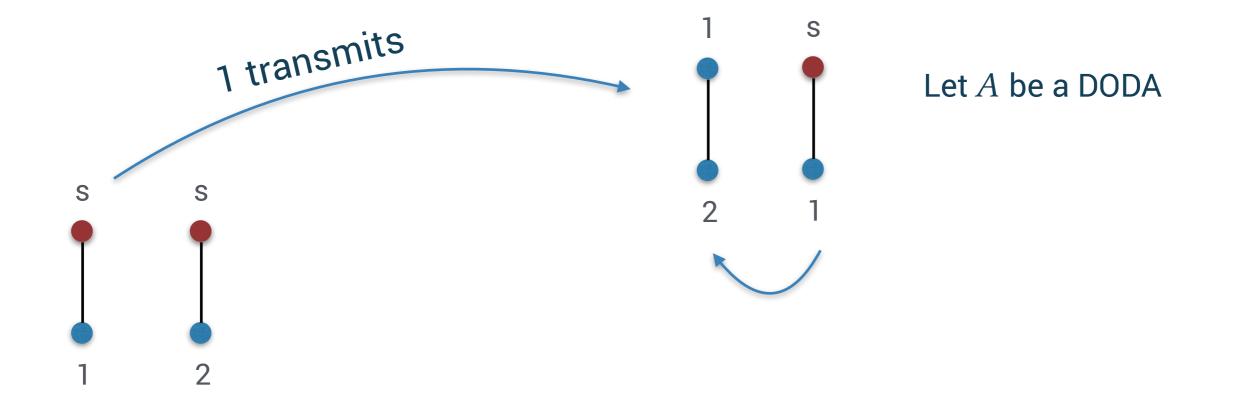


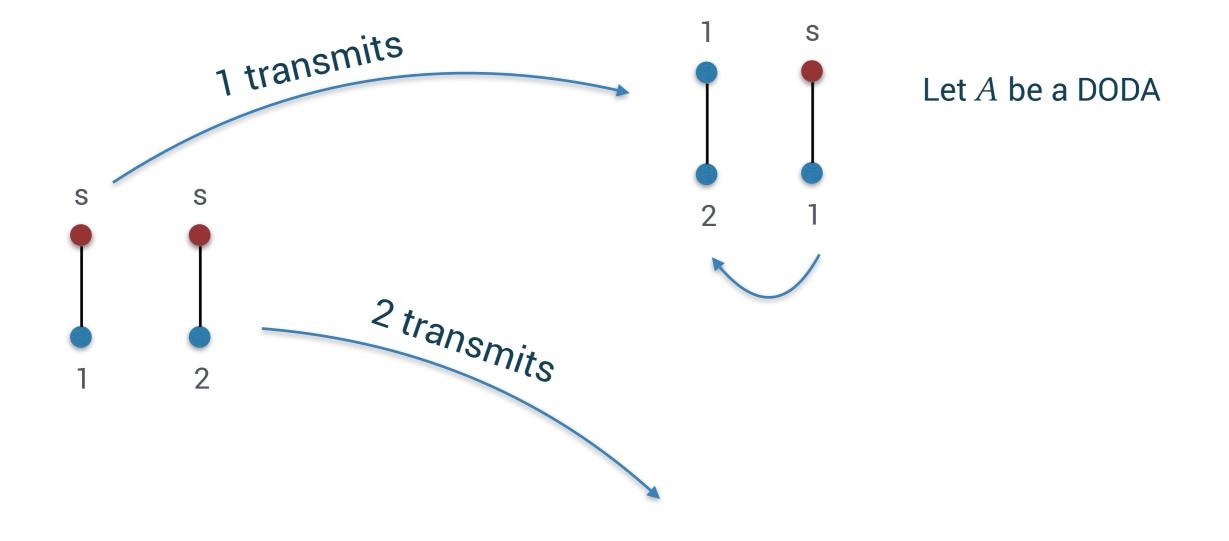
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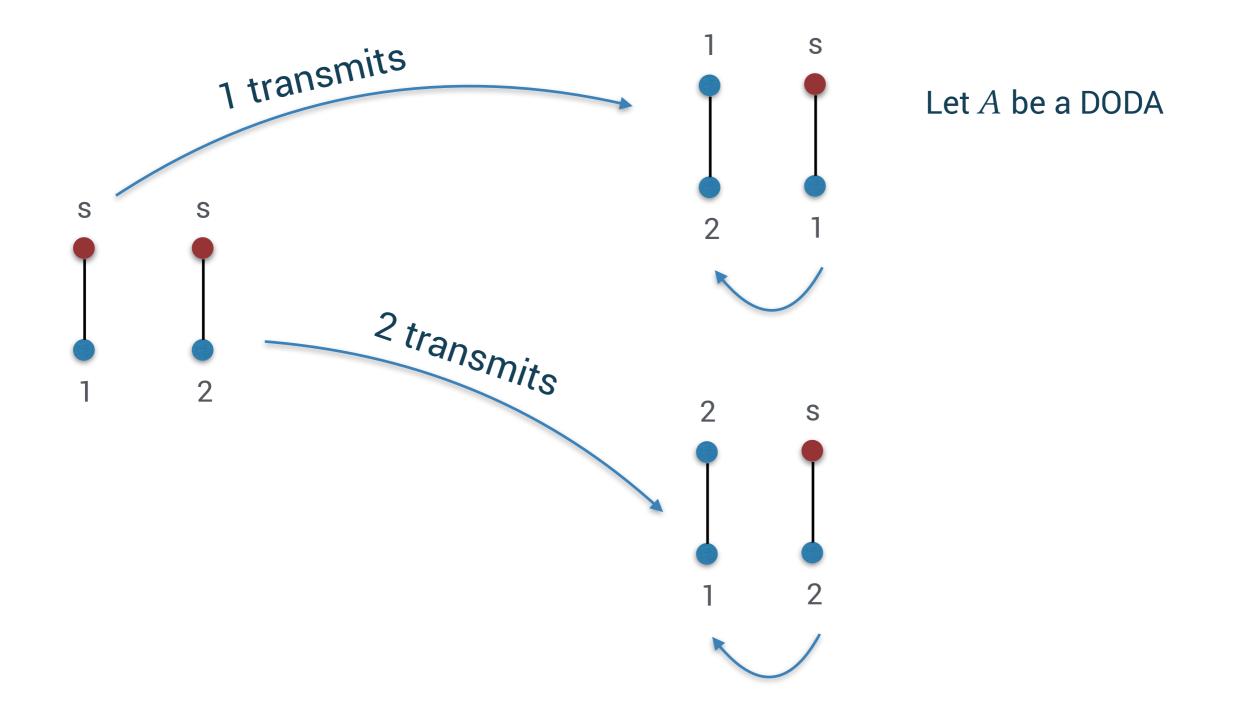


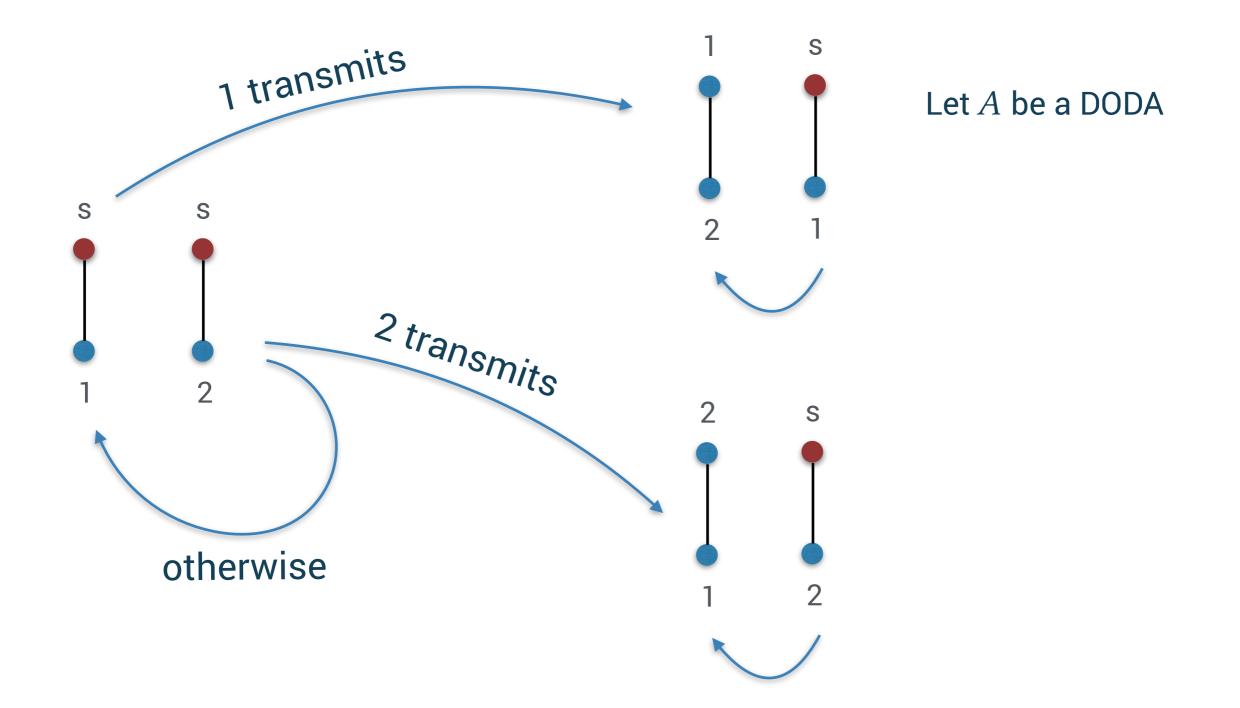




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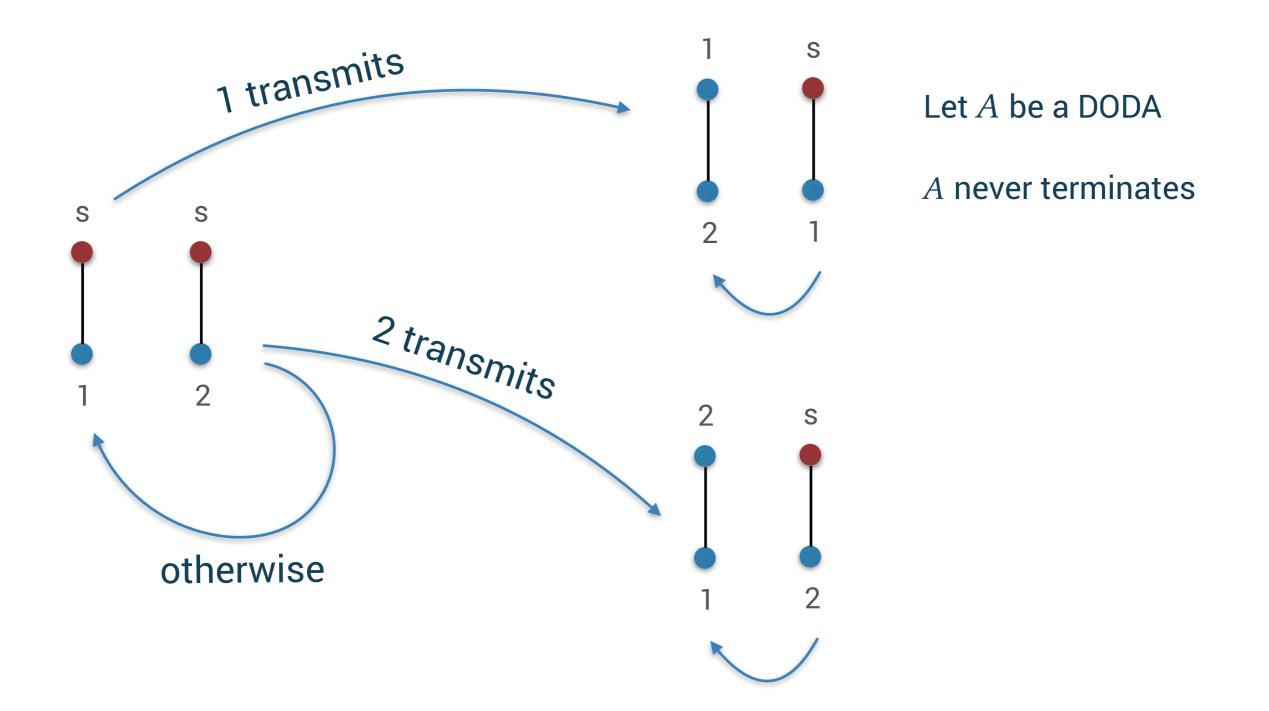
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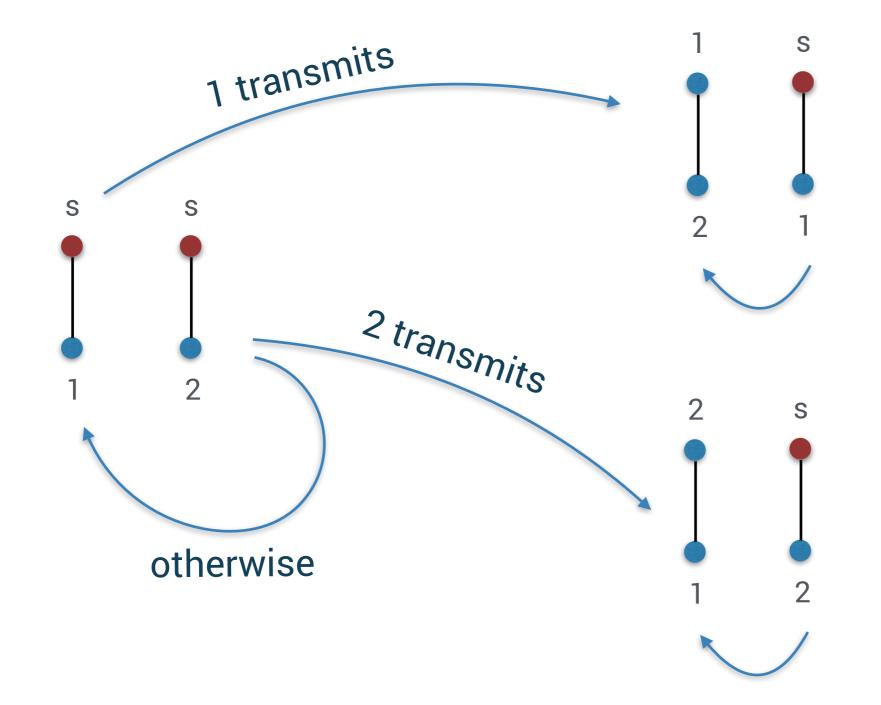


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27



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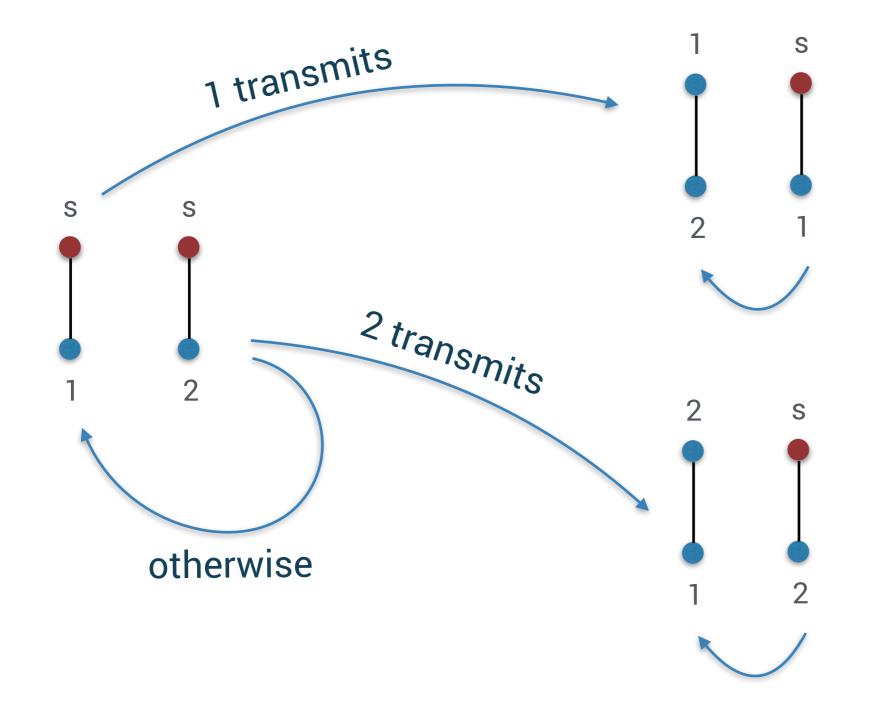
Let A be a DODA

A never terminates

Starting from any time t, the aggregation is always possible, so:

27

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A never terminates

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 $cost(A) = \infty$ 

28

**Theorem:** If k transmissions are allowed per node, there exists an online adaptive adversary that generates for every DODA A a sequence I such that  $cost_I(A) = \infty$ 

28

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**Corollary:** If k transmissions are allowed per node, there exists an **oblivious** adversary that generates for every **deterministic** DODA *A* a sequence *I* such that  $cost_I(A) = \infty$ 

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What about randomized DODA algorithms against an oblivious adversary?

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**Theorem:** If k transmissions are allowed per node, there exists an oblivious adversary that generates for every **oblivious** DODA A a sequence I such that  $cost_I(A) = \infty$  with high probability

What about randomized DODA algorithms against an oblivious adversary?

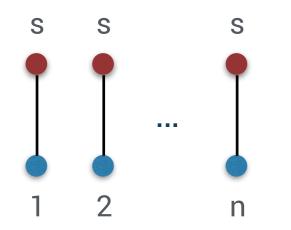
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proof: not trivial

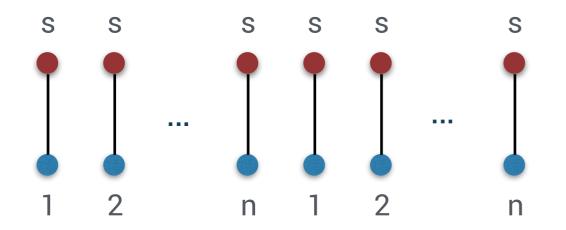
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**sketch of proof**, for k=1

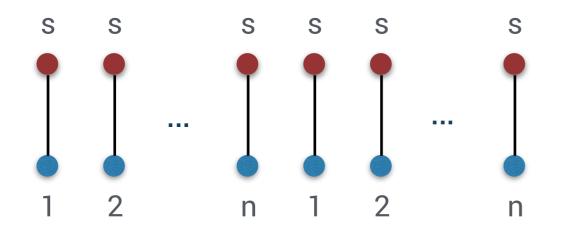
#### sketch of proof, for k=1



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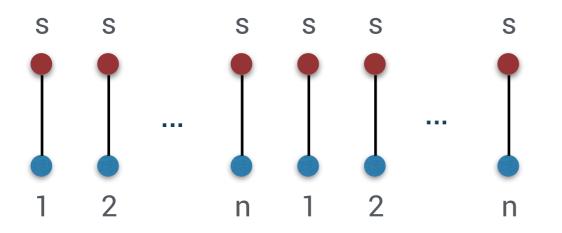


#### sketch of proof, for k=1



 $P_t$  = probability that no node transmits before time t

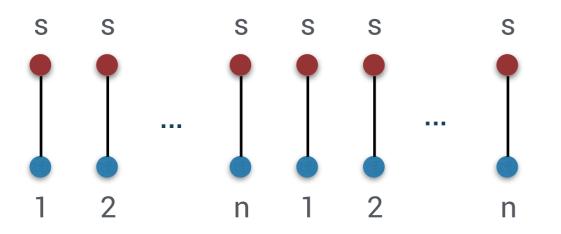
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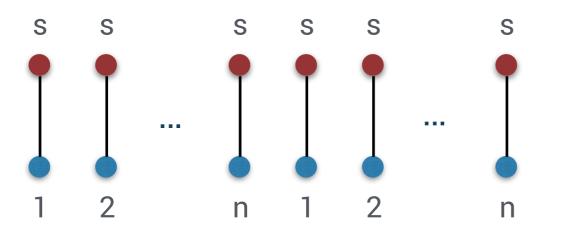


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if  $P_t > 1/n$  for all *t*, then it's over

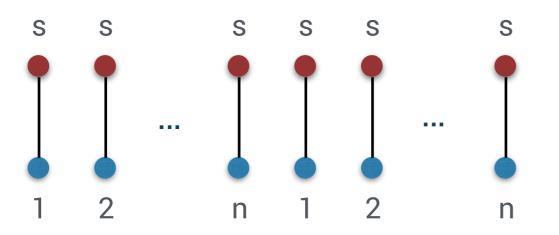
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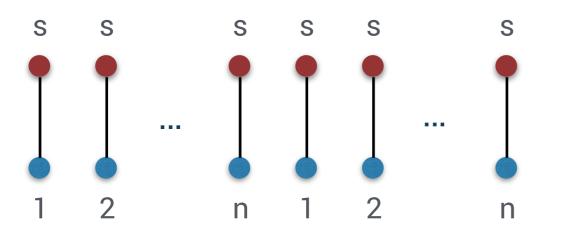


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- $P_{t0-1} > 1/n$
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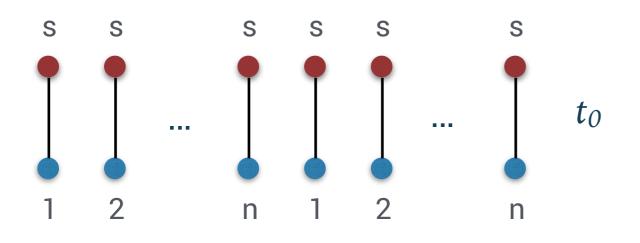
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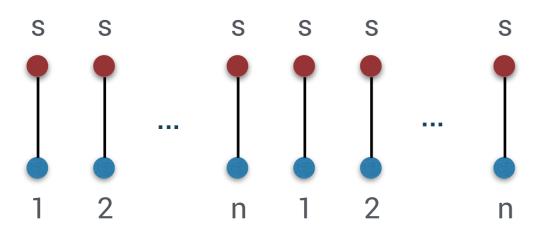
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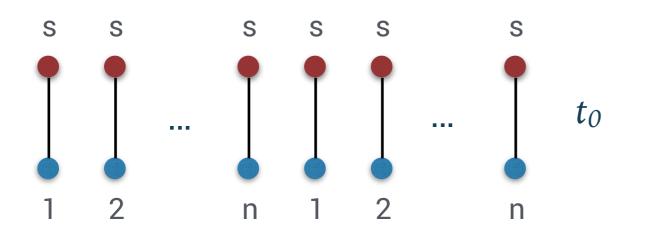
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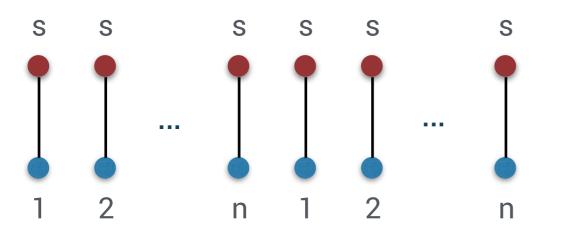
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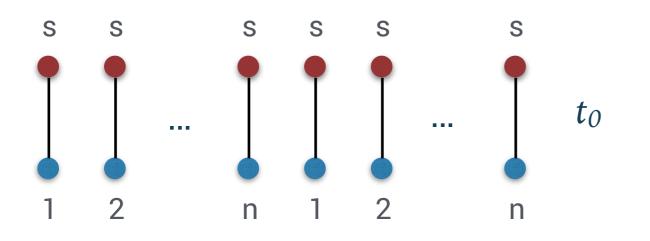
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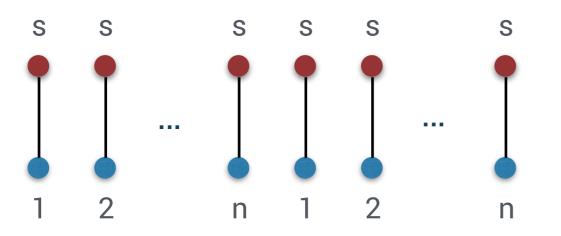
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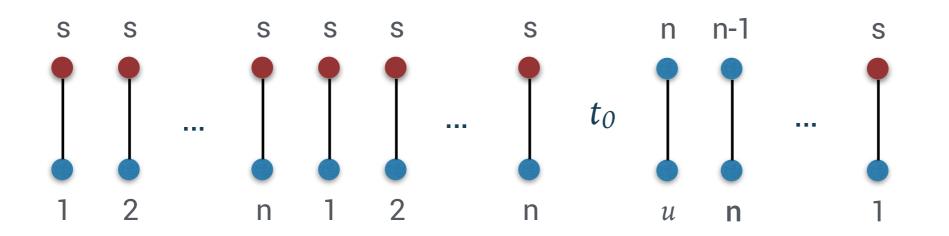
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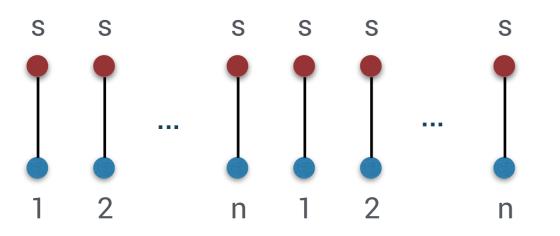
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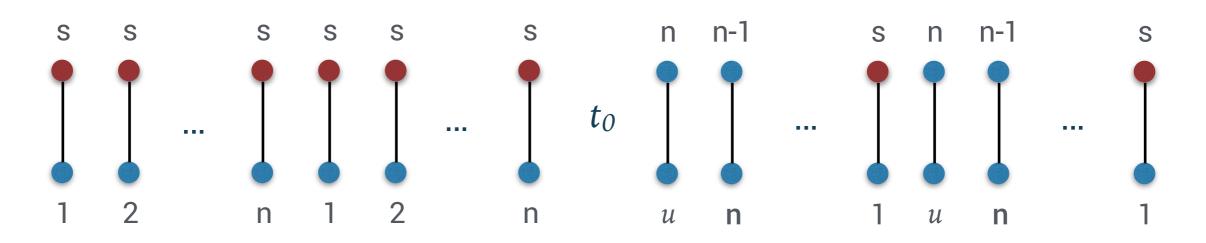
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**sketch of proof**, for k>1

**I**k-1

*I<sup>k-1</sup>* Each group of nodes acts like a node
that can transmit only once

**I**k-1

require n<sup>k</sup> nodes

What about randomized DODA algorithms against oblivious adversary?

**Theorem:** If k transmissions are allowed per node, there exists an oblivious adversary that generate for every **oblivious** DODA A a sequence I such that  $cost_I(A) = \infty$  with high probability

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What about non-oblivious randomized DODA algorithms against oblivious adversary?

open question

If you know everything about the future, it takes O(nlog(n)) interactions in average.

If you have no information about the future, always transmit is optimal. It takes  $O(n^2)$  interactions in average.

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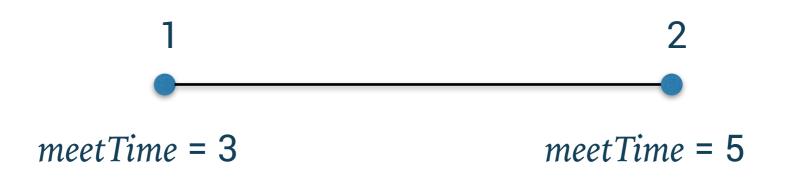
The *meetTime* information: each node knows when will be its next interaction with the sink

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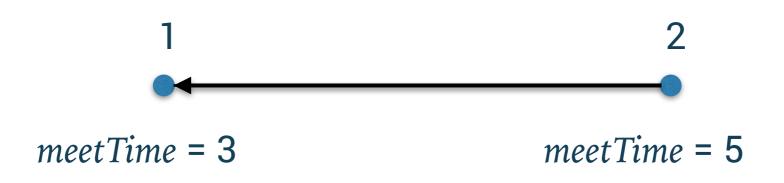


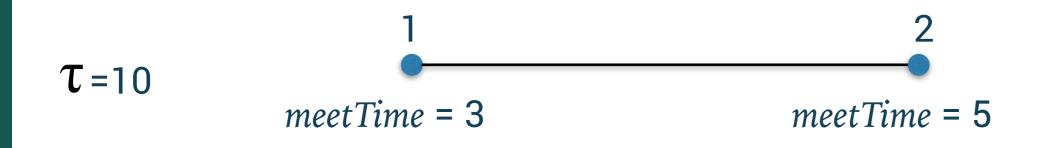
Greedy Algorithm: **if** I meet the sink after the other node, **then** I transmit my data

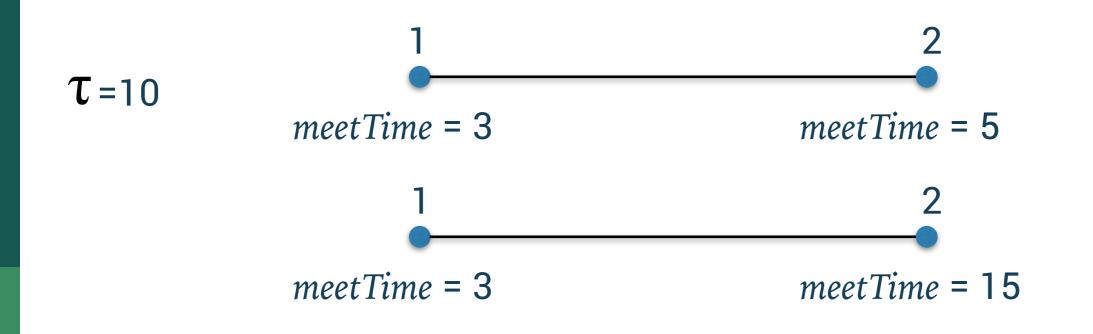
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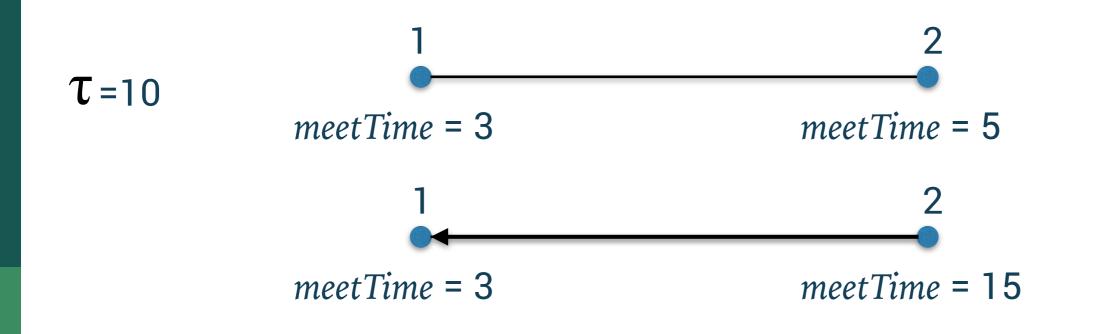


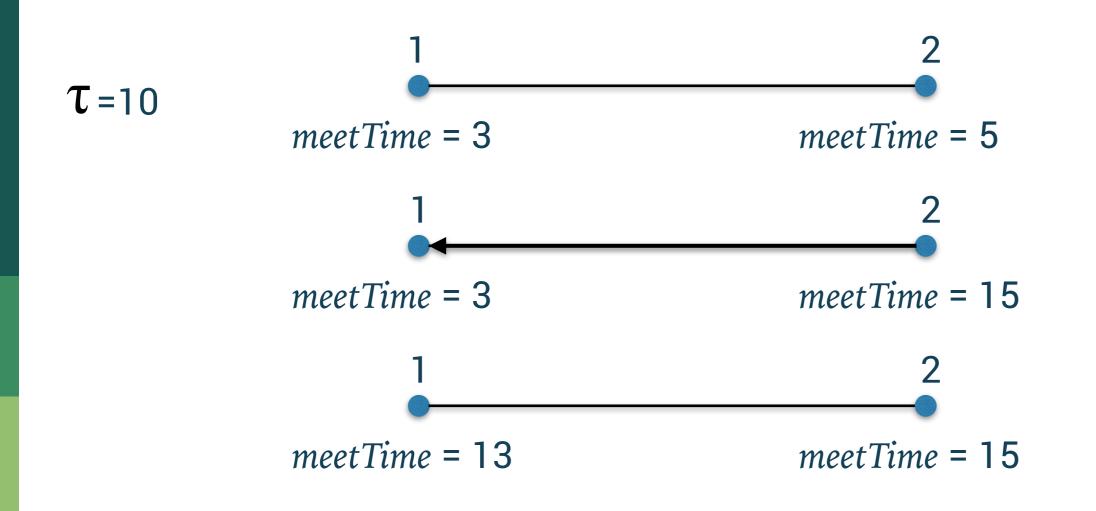
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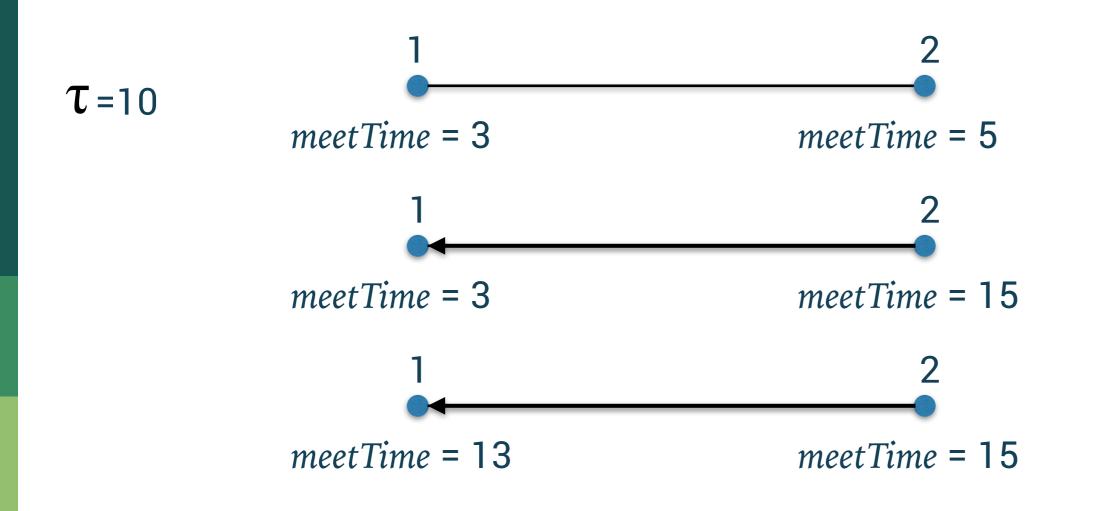


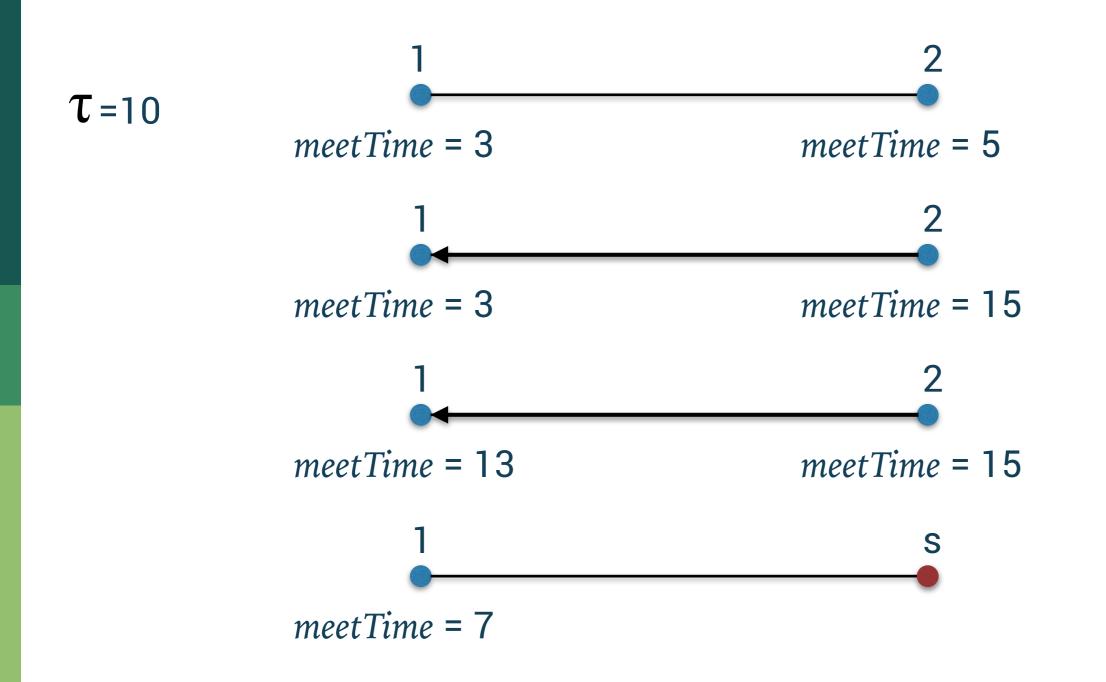












 $\tau$ -Waiting Greedy Algorithm: **if** at least one of the nodes has *meetTime* >  $\tau$ , **then** we apply Greedy Algorithm

 $n\sqrt{n \log(n)}$ -Waiting Greedy Algorithm is optimal

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interactions in average

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Without knowledge:

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With full knowledge:

 $\Theta(n \log(n))$  interactions w.h.p.

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 $n\sqrt{n \log(n)}$ -Waiting Greedy Algorithm is optimal

Without knowledge: $\Theta(n^2)$ interactions in averagemeetTime information: $\Theta(n\sqrt{n \log(n)})$ interactions w.h.p.With full knowledge: $\Theta(n \log(n))$ interactions w.h.p.

Online Data Aggregation:

- hard in general
- optimal algorithms exist in randomized networks

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## Perspective

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More realistic networks?

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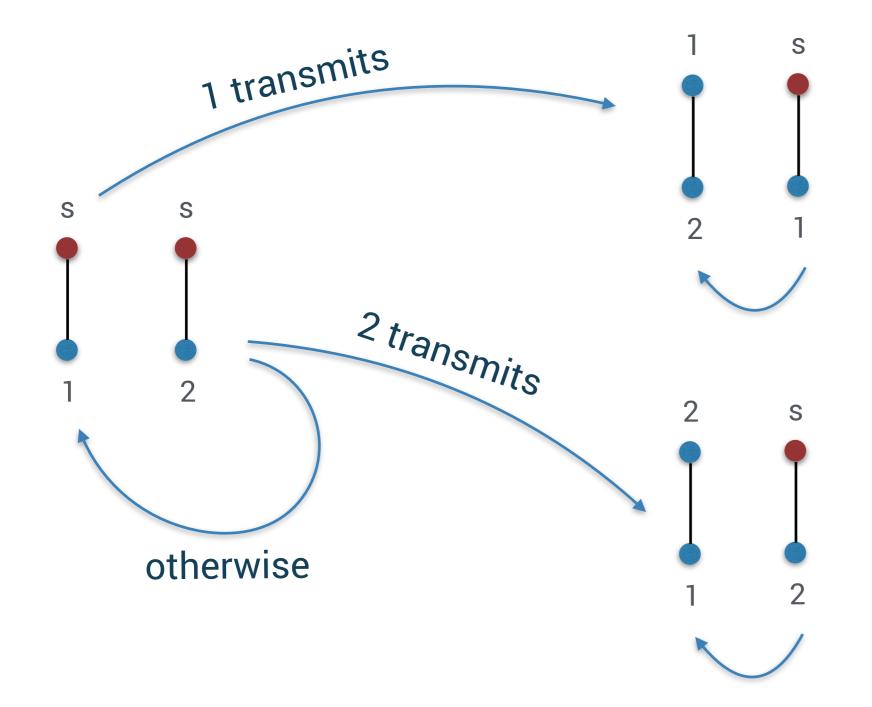
## Perspective

More realistic networks?

Thank you for your attention!

quentin.bramas@lip6.fr

#### What if nodes are allowed to transmit more than once?

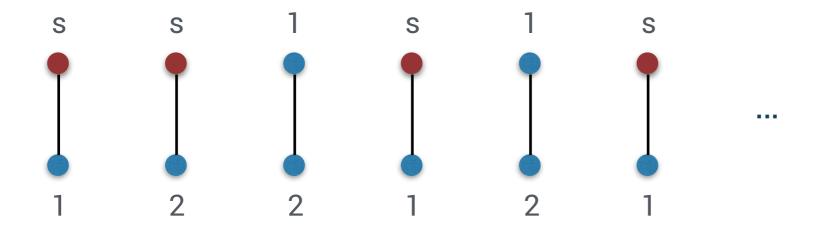


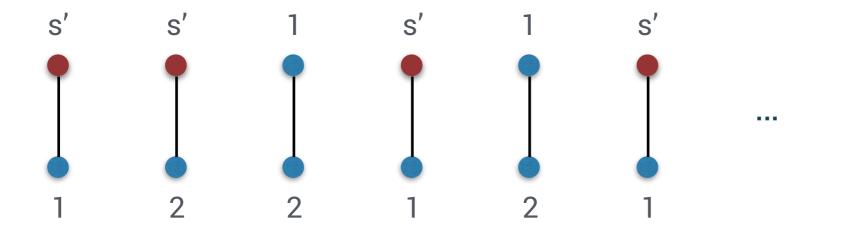
#### Let *A* be a DODA

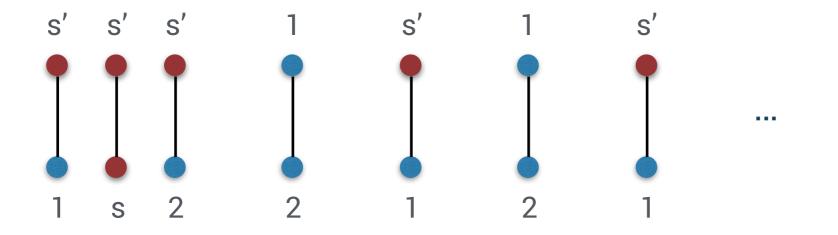
Starting from any time t, the aggregation is always possible, so:

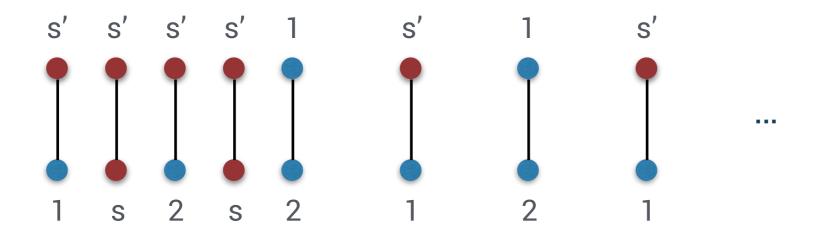
41

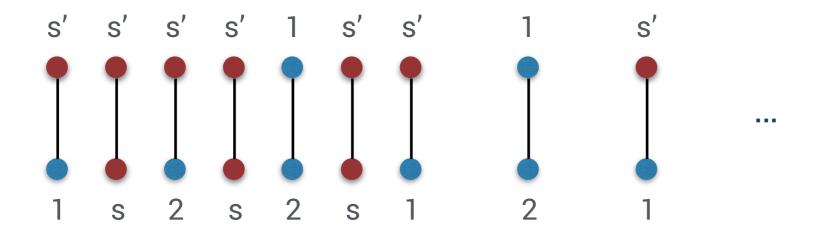
 $cost(A) = \infty$ 

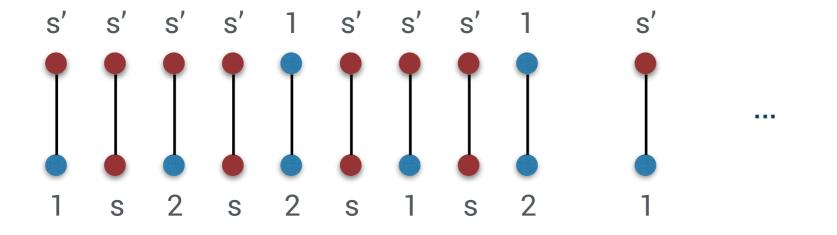


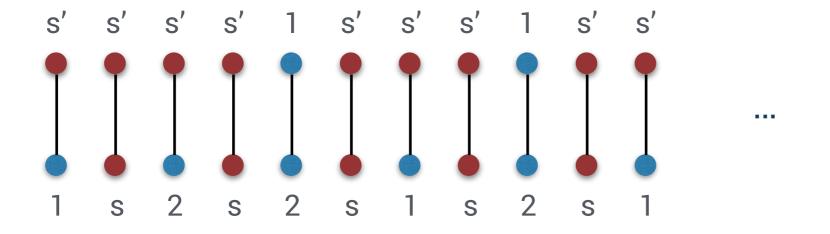


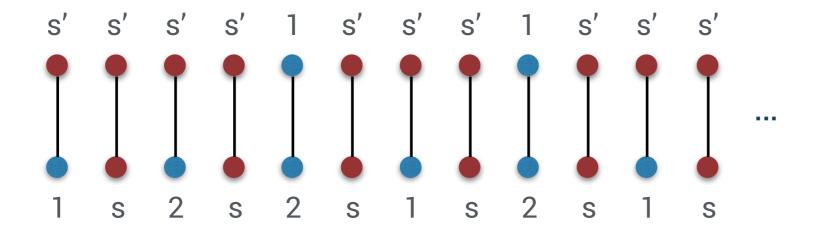


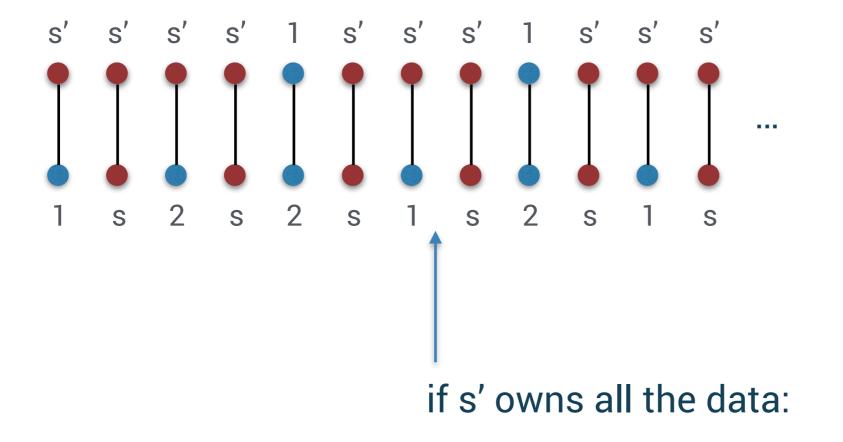




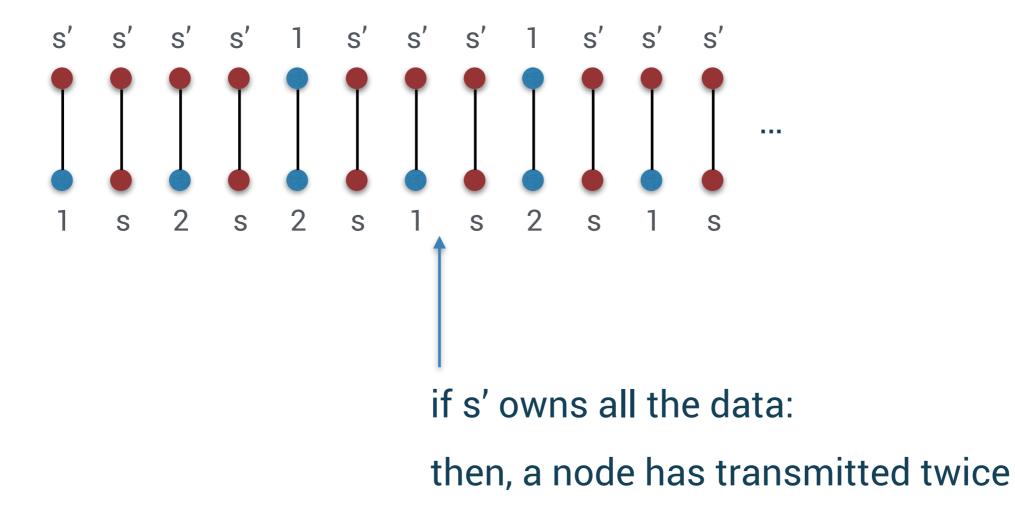


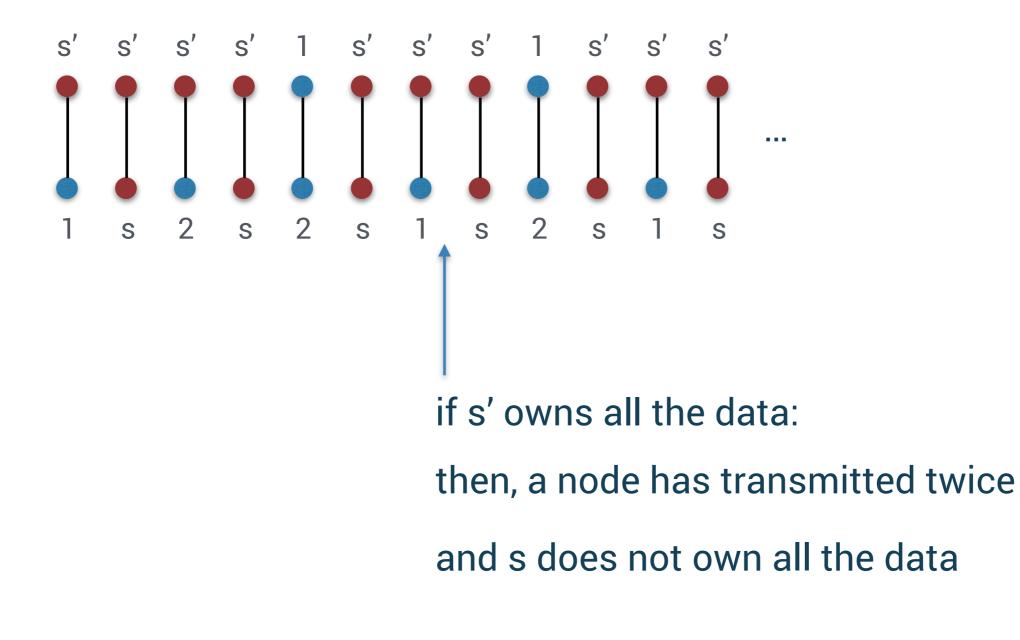




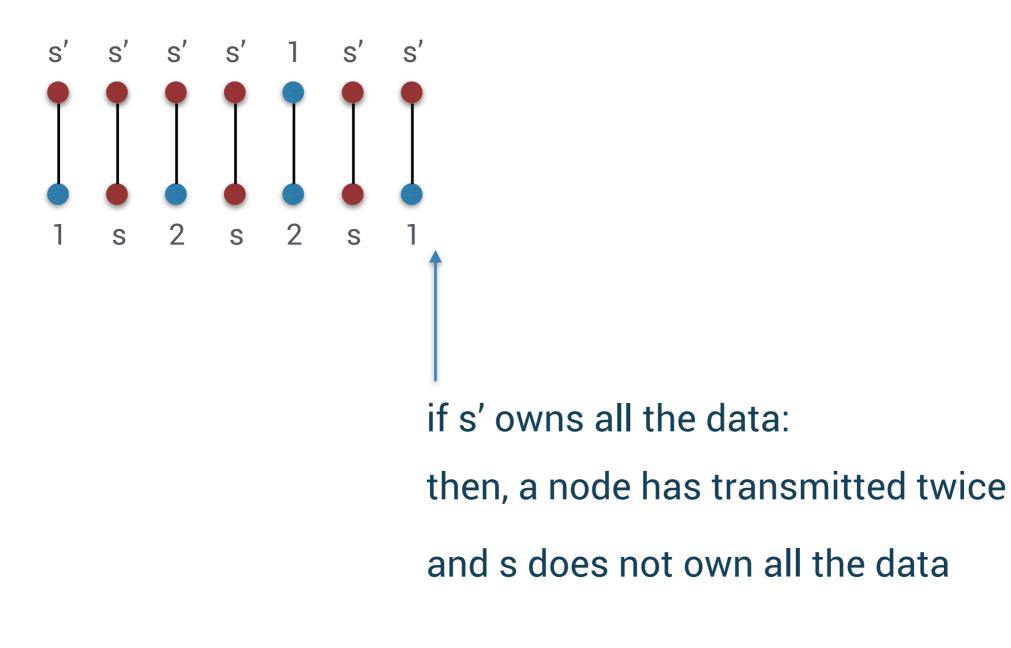


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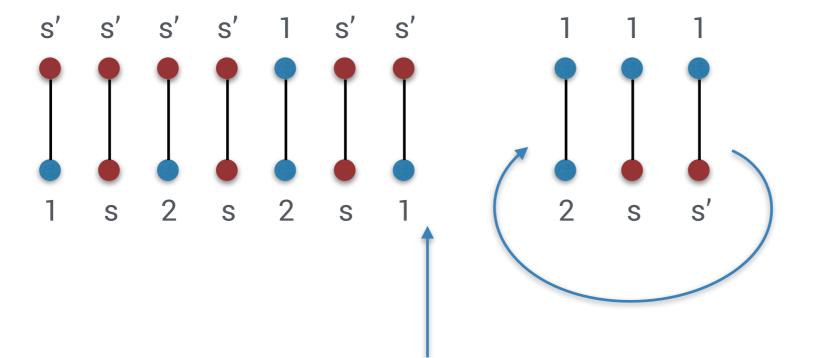




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#### What if nodes are allowed to transmit more than once?

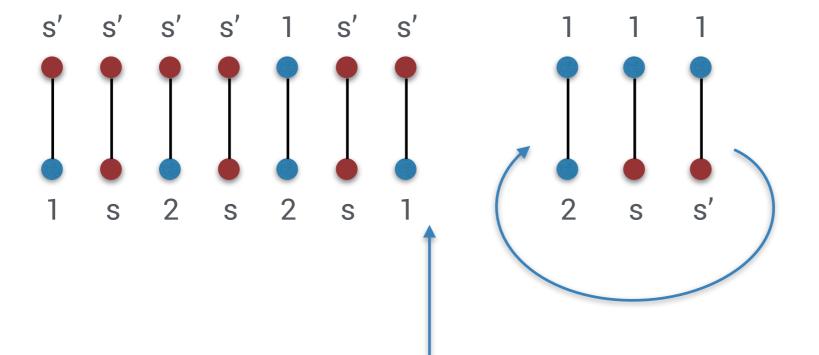


if s' owns all the data:

then, a node has transmitted twice

and s does not own all the data

#### What if nodes are allowed to transmit more than once?

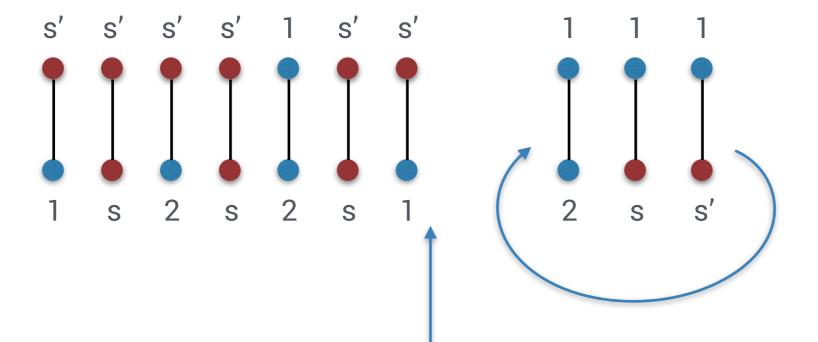


if s' owns all the data:

then, a node has transmitted twice

and s will never own all the data

#### What if nodes are allowed to transmit more than once?



if s' owns all the data:

then, a node has transmitted twice

and s will never own all the data yet a convergecast is always possible